2.4 Newton's Second Law of Motion

In recent tests of different vehicles, the minimum time for each one to accelerate from zero to 96.5 km/h [fwd] was measured. The results are shown in **Table 1**. It is evident from the table that the acceleration of a vehicle depends greatly on the power of the engine causing the acceleration and the mass of the vehicle. Logically, the magnitude of acceleration increases as the engine power increases and decreases as the mass increases.

Table 1 Data for Vehicles Accelerating from Zero to 96.5 km/h (60.0 mi/h)

Vehicle	Mass (kg)	Engine power (kW)	Time to accelerate (s)
Chevrolet Camaro	1500	239	5.5
Rolls-Royce	2300	240	8.2
Cadillac Seville	1800	224	6.8
Lamborghini Diablo	1800	410	3.6
Volkswagen Beetle	1260	86	10.6
Suzuki Bandit Bike	245	74	2.8

You observed a similar relationship in Investigation 2.3.1, although the variables you tested were net force and mass. These observations led to Newton's second law of motion.

Newton's first law of motion deals with situations in which the net external force acting on an object is zero, so no acceleration occurs. His second law deals with situations in which the net external force acting on an object is not zero, so acceleration occurs in the direction of the net force. The acceleration increases as the net force increases, but decreases as the mass of the object increases.

Newton's **second law of motion** states:

second law of motion: if the net external force on an object is not zero, the object accelerates in the direction of the net force, with magnitude of acceleration proportional to the magnitude of the net force and inversely proportional to the object's mass

Second Law of Motion

If the net external force on an object is not zero, the object accelerates in the direction of the net force. The magnitude of the acceleration is proportional to the magnitude of the net force and is inversely proportional to the object's mass.

Using mathematical notation, we can derive an equation for the second law of motion.

$\vec{a} \propto \vec{F}_{net}$ when m is constant

$\vec{a} \propto \dfrac{1}{m}$ when $\vec{F}_{net}$ is constant

Thus, $\vec{a} \propto \dfrac{\vec{F}_{net}}{m}$.

Now, we insert a proportionality constant, k, to create the equation relating all three variables:

$$\vec{a} = k\frac{\vec{F}_{net}}{m}$$

If the units on both sides of the equation are consistent, with newtons for force and the preferred SI units of metres, kilograms, and seconds for the acceleration and mass, the value of k is 1. Thus, the final equation for Newton's second law of motion is

$\vec{a} = \dfrac{\vec{F}_{net}}{m}$ Acceleration equals net force divided by mass.

This equation is often written in rearranged form:

$\vec{F}_{net} = m\vec{a}$ Net force equals mass times acceleration.

$$\vec{F}_{net} = m\vec{a}$$

As stated earlier, the unit for force is the newton. We can use the second law equation to define the newton in terms of the SI base units.

Therefore, 1 N = 1 kg•m/s^2.

Thus, we can define *one newton* as the magnitude of the net force required to give a 1.0 kg object an acceleration of magnitude 1.0 m/s^2.

Newton's second law of motion is important in physics. It affects all particles and objects in the universe. Because of its mathematical nature, the law is applied in finding the solutions to many problems and questions.

Sample Problem 1

A net force of 58 N [W] is applied to a water polo ball of mass 0.45 kg. Calculate the ball's acceleration.

Solution

$\vec{F}_{net} = 58 \text{ N [W]}$

$m = 0.45 \text{ kg}$

$\vec{a} = ?$

$$\begin{aligned}\vec{a} &= \frac{\vec{F}_{net}}{m} \\ &= \frac{58 \text{ N [W]}}{0.45 \text{ kg}} \\ &= \frac{58 \text{ kg•m/s}^2 \text{ [W]}}{0.45 \text{ kg}} \\ \vec{a} &= 1.3 \times 10^2 \text{ m/s}^2 \text{ [W]}\end{aligned}$$

The ball's acceleration is 1.3×10^2 m/s^2 [W].

Sample Problem 2

In an extreme test of its braking system under ideal road conditions, a Toyota Celica, travelling initially at 26.9 m/s [S], comes to a stop in 2.61 s. The mass of the car with the driver is 1.18×10^3 kg. Calculate (a) the car's acceleration and (b) the net force required to cause that acceleration.

Solution

(a) $\vec{v}_f = 0.0$ m/s [S]

$\vec{v}_i = 26.9$ m/s [S]

$\Delta t = 2.61$ s

$\vec{a} = ?$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$= \frac{0.0 \text{ m/s [S]} - 26.9 \text{ m/s [S]}}{2.61 \text{ s}}$$

$$\vec{a} = -10.3 \text{ m/s}^2 \text{ [S]}$$

The car's acceleration is -10.3 m/s^2 [S], or 10.3 m/s^2 [N].

(b) $m = 1.18 \times 10^3$ kg

$\vec{F}_{net} = ?$

$$\vec{F}_{net} = m\vec{a}$$

$$= (1.18 \times 10^3 \text{ kg})(10.3 \text{ m/s}^2 \text{ [N]})$$

$$\vec{F}_{net} = 1.22 \times 10^4 \text{ N [N]}$$

The net force is 1.22×10^4 N [N].

Does Newton's second law agree with his first law of motion? According to the second law, $\vec{a} = \frac{\vec{F}_{net}}{m}$, so the acceleration is zero when the net force is zero. This is in exact agreement with the first law. In fact, the first law is simply a special case ($\vec{F}_{net} = 0$) of the second law of motion.

Practice

Understanding Concepts

1. Calculate the acceleration in each situation.
   (a) A net force of 27 N [W] is applied to a cyclist and bicycle having a total mass of 63 kg.
   (b) A bowler exerts a net force of 18 N [fwd] on a 7.5-kg bowling ball.
   (c) A net force of 32 N [up] is applied to a 95-g model rocket.
2. Find the magnitude and direction of the net force in each situation.
   (a) A cannon gives a 5.0-kg shell a forward acceleration of 5.0×10^3 m/s^2 before it leaves the muzzle.
   (b) A 28-g arrow is given an acceleration of 2.5×10^3 m/s^2 [E].
   (c) A 500-passenger Boeing 747 jet (with a mass of 1.6×10^5 kg) undergoes an acceleration of 1.2 m/s^2 [S] along a runway.
3. Write an equation expressing the mass of an accelerated object in terms of its acceleration and the net force causing that acceleration.
4. Determine the mass of a regulation shot in the women's shot-put event (**Figure 1**) if a net force of 7.2×10^2 N [fwd] is acting on the shot, giving the shot an average acceleration of 1.8×10^2 m/s^2 [fwd].
5. Derive an equation for net force in terms of mass, final velocity, initial velocity, and time.
6. Assume that during each pulse a mammalian heart accelerates 21 g of blood from 18 cm/s to 28 cm/s during a time interval of 0.10 s. Calculate the magnitude of the force (in newtons) exerted by the heart muscle on the blood.

Answers

1. (a) 0.43 m/s^2 [W]
   (b) 2.4 m/s^2 [fwd]
   (c) 3.4×10^2 m/s^2 [up]
2. (a) 2.5×10^4 N [fwd]
   (b) 7.0×10^1 N [E]
   (c) 1.9×10^5 N [S]
4. 4.0 kg
6. 2.1×10^{-2} N

Figure 1
For question 4

SUMMARY Newton's Second Law of Motion

- Newton's second law of motion relates the acceleration of an object to the mass of the object and the net force acting on it. The equation is

$$\vec{a} = \frac{\vec{F}_{net}}{m} \quad \text{or} \quad \vec{F}_{net} = m\vec{a}.$$

- Newton's second law is applied in many problem-solving situations.

Section 2.4 Questions

Understanding Concepts

1. When the Crampton coal-fired train engine was built in 1852, its mass was 48.3 t (1.0 t = 1.0×10^3 kg) and its force capability was rated at 22.4 kN. Assuming it was pulling train cars whose total mass doubled its own mass and the total friction on the engine and cars was 10.1 kN, what was the magnitude of the acceleration of the train?
2. Determine the net force needed to cause a 1.31×10^3-kg sports car to accelerate from zero to 28.6 m/s [fwd] in 5.60 s.
3. As you have learned from Chapter 1, the minimum safe distance between vehicles on a highway is the distance a vehicle can travel in 2.0 s at a constant speed. Assume that a 1.2×10^3-kg car is travelling 72 km/h [S] when the truck ahead crashes into a northbound truck and stops suddenly.
   (a) If the car is at the required safe distance behind the truck, what is the separation distance?
   (b) If the average net braking force exerted by the car is 6.4×10^3 N [N], how long would it take the car to stop?
   (c) Determine whether a collision would occur. Assume that the driver's reaction time is an excellent 0.09 s.

2.5 Newton's Third Law of Motion

Figure 1
The astronaut is wearing a manoeuvring unit (the backpack) that illustrates an application of the third law of motion. Expanding gases expelled from the unit propel the astronaut in a direction opposite to the direction of the expelled gases.

When astronauts go for a "space walk" outside the International Space Station (the ISS), they travel along with the station at a speed of about 30 000 km/h relative to Earth's surface. (You should be able to use the first law of motion to explain why: since the station and the astronaut are both in motion, they remain in motion together.) However, to move around outside the station to make repairs, the astronaut must be able to manoeuvre in different directions relative to the station. To do so, the astronaut wears a special backpack called a mobile manoeuvring unit, or MMU (**Figure 1**), a device that applies another important principle named after Sir Isaac Newton.

Newton's first law of motion is descriptive and his second law is mathematical. In both cases, we consider the forces acting on only one object. However, when your hand pushes on the desk in one direction, you feel a force of the desk pushing back on your hand in the opposite direction. This brings us to the third law, which considers forces acting in pairs on two objects.