9. Often a battery from a second vehicle is connected to the discharged battery of a vehicle that won't start in order to recharge or "boost" the battery. Should the batteries be connected in series or parallel? Explain your answer. What terminals should be connected?

Reflecting

10. When dealing with electric circuits, safety should come first. How does the study of electric circuits help people to become more aware of proper safety rules for circuits? List three things that could be done to make the use of electric circuits safer.

12.6 Electric Resistance

resistance: an opposition to the flow of charge, resulting in a loss of potential energy

When charges pass through a material or device, they experience an opposition or **resistance** to their flow, resulting in a loss of electric potential energy. To measure the amount of resistance that a quantity of moving charge encounters, we compare the electric potential difference the charge experiences as it passes through a conductor with the amount of electric current.

The circuit in **Figure 1** shows a resistance and a source of variable potential difference. The ammeter indicates the current flowing through the resistance, while the voltmeter indicates the potential difference across the resistance.

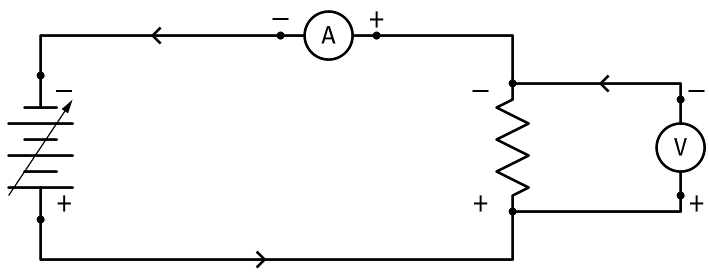

Figure 1

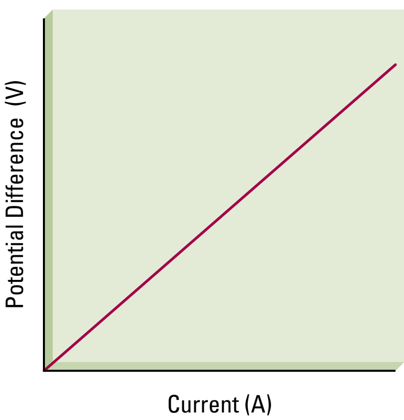

Figure 2
Mathematically, this graph is of the form $y = mx$, where m is the slope of the straight line.

By varying the potential difference of the source and making simultaneous measurements of current and potential difference, the graph in **Figure 2** can be obtained.

Ohm's Law

The German physicist Georg Simon Ohm (1787–1854), shown in **Figure 3**, found that, for a given conductor, the ratio $\frac{V}{I}$ is a constant. From this constant ratio, he formulated what we now call **Ohm's law**:

Ohm's law: The potential difference between any two points in a conductor varies directly as the current between the two points if the temperature remains constant.

Ohm's Law

The potential difference between any two points in a conductor varies directly as the current between the two points if the temperature remains constant.

This relationship can be written as

$$\frac{V}{I} = \text{constant}$$

Since the constant depends on the properties of the particular resistor being used, we give it the symbol R and call it resistance. Therefore, Ohm's law can be written as:

$$\frac{V}{I} = R$$

V is measured in volts, and I is measured in amperes, so R is measured in volts per ampere, and this new unit, the unit of electric resistance, is called the **ohm** (Ω).

1 Ω is the electric resistance of a conductor that has a current of 1 A through it when the potential difference across it is 1 V.

$1\ \Omega = 1\ \text{V/A}$

Figure 3
Georg Simon Ohm (1787–1854) became a high school teacher but longed to work at a university. To secure a university post, he began to do research in the area of electrical conduction. After his discovery of the law that bears his name, he received so much public criticism that he was forced to resign even his high school position. His fellow scientists felt that his discoveries were based too much on theory and lacked the experimental proof to make them acceptable. After years of living in poverty, he was finally recognized by the Royal Society, which conferred membership on him in 1842. He was also awarded a professorship at the University of Munich, where he spent the last five years of his life with his ambition realized.

ohm: (Ω) the SI unit for electric resistance

Sample Problem 1

What is the potential difference across a toaster of resistance 13.8 Ω when the current through it is 8.7 A?

Solution

$R = 13.8\ \Omega$

$I = 8.7\ \text{A}$

$$\begin{aligned} V &= IR \\ &= (8.7\ \text{A})\ (13.8\text{V}) \\ V &= 1.2 \times 10^2\ \text{V} \end{aligned}$$

The potential difference across the toaster is 1.2×10^2 V.

Practice

Understanding Concepts

1. Calculate the value of the resistance in each case.
   (a) $V = 12$ V, $I = 0.25$ A
   (b) $V = 1.50$ V, $I = 30.0$ mA
   (c) $V = 2.4 \times 10^4$ V, $I = 6.0 \times 10^{-3}$ A
2. Calculate the maximum rating (in volts) of a battery used to operate a toy electric motor that has a resistance of 2.4 Ω and runs at top speed with a current of 2.5 A.
3. Write an equation for electric current in terms of electric potential and resistance.
4. Find the unknown quantities.
   (a) $R = 35\ \Omega$, $I = 0.45$ A, $V = ?$ (b) $R = 2.2$ kΩ, $I = 1.5$ A, $V = ?$
   (c) $V = 6.0$ V, $R = 18\ \Omega$, $I = ?$ (d) $V = 52$ mV, $R = 26\ \Omega$, $I = ?$
5. What current is drawn by a vacuum cleaner from a 115-V circuit having a resistance of 28 Ω?
6. A walkie-talkie receiver operates on a 9.0-V battery. If the receiver draws 3.0×10^2 mA of current, what is the resistance?

Answers

1. (a) 48 Ω
   (b) 50.0 Ω
   (c) $4.0 \times 10^6\ \Omega$
2. 6.0 V
4. (a) 16 V
   (b) 3.3×10^3 V
   (c) 0.33 A
   (d) 2.0×10^{-3} A
5. 4.1 A
6. $3.0 \times 10^1\ \Omega$

Resistance in Series

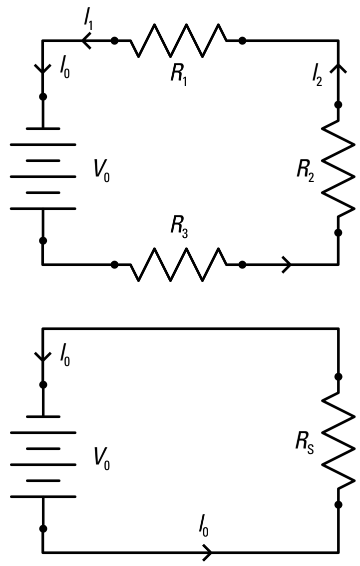

Figure 4

Figure 4 shows a circuit with one source of electric potential, V_0, and three resistors, R_1, R_2, and R_3, connected in series as indicated. We would like to find the value of the **equivalent resistor**, R_S, so the circuit would simply contain the source, V_0, and the resistor, R_S. The amount of current through the circuit from the source is I_0.

equivalent resistor: resistor that has the same current and potential difference as the resistors it replaces

Applying Kirchhoff's voltage law to the circuit,

$$(1)\quad V_0 = V_1 + V_2 + V_3$$

Applying Ohm's law to each individual resistor,

$$(2)\quad V_1 = I_1R_1 \qquad V_2 = I_2R_2 \qquad V_3 = I_3R_3 \qquad V_0 = I_0R_S$$

Substituting equation (2) into (1),

$$I_0R_S = I_1R_1 + I_2R_2 + I_3R_3$$

Applying Kirchhoff's current law to the circuit,

$$I_0 = I_1 = I_2 = I_3$$

Therefore, as an expression for the equivalent resistor of R_1, R_2, and R_3, in series, we get

$$R_S = R_1 + R_2 + R_3$$

And if the number of resistors connected in series is n, the equivalent resistor will be given by

$$R_S = R_1 + R_2 + \ldots + R_n$$

Sample Problem 2

What is the equivalent resistor in a series circuit containing a 16 Ω light bulb, a 27 Ω heater, and a 12 Ω motor?

Solution

$R_1 = 16\ \Omega$
$R_2 = 27\ \Omega$
$R_3 = 12\ \Omega$
$R_S = ?$

$$\begin{aligned} R_S &= R_1 + R_2 + R_3 \\ &= 16\ \Omega + 27\ \Omega + 12\ \Omega \\ R_S &= 55\ \Omega \end{aligned}$$

The equivalent resistor is 55 Ω.

Practice

Understanding Concepts

7. Find the equivalent resistance in each of these cases.
   (a) A 12-Ω, a 25-Ω, and a 42-Ω resistor are connected in series.
   (b) Two 30-Ω light bulbs and two 20-Ω heating elements are connected in series.
   (c) Two strings of Christmas tree lights are connected in series; the first string has eight 4.0-Ω bulbs in series, and the second has twelve 3.0-Ω bulbs in series.
8. Find the value of the unknown resistance in each of these cases.
   (a) A 22-Ω, an 18-Ω, and an unknown resistor are connected in series to give an equivalent resistance of 64 Ω.
   (b) Two identical unknown bulbs are connected in series with a 48-Ω and a 64-Ω heater to produce an equivalent resistance of 150 Ω.
   (c) Each light in a series string of 24 identical bulbs has an equivalent resistance of 48 Ω.

Answers

7. (a) 79 Ω
   (b) 100 Ω
   (c) 68 Ω
8. (a) 24 Ω
   (b) 19 Ω
   (c) 2.0 Ω

Resistance in Parallel

We can use the same approach to find the equivalent resistance of several resistors connected in parallel (**Figure 5**). If we call the equivalent resistance of R_1, R_2, and R_3 connected in parallel R_P, then the circuit appears as shown.

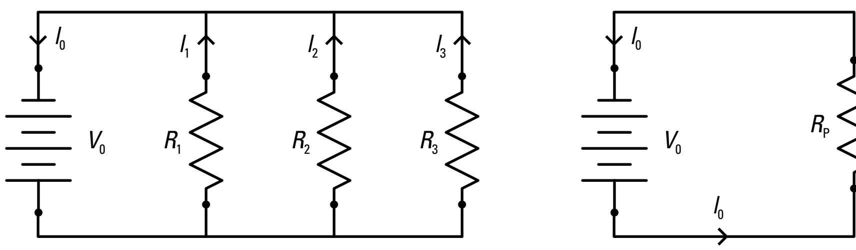

Figure 5

Applying Kirchhoff's current law to the circuit,

$$(1)\quad I_0 = I_1 + I_2 + I_3$$

Applying Ohm's law to each individual resistance,

$$(2)\quad I_1 = \frac{V_1}{R_1} \qquad I_2 = \frac{V_2}{R_2} \qquad I_3 = \frac{V_3}{R_3} \qquad I_0 = \frac{V_0}{R_P}$$

Substituting equation (2) into (1),

$$\frac{V_0}{R_P} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

Applying Kirchhoff's voltage law to the circuit,

$$V_0 = V_1 = V_2 = V_3$$

Therefore, as an expression for the equivalent resistance of R_1, R_2, and R_3, in parallel, we get

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

And, if the number of resistors connected in parallel is n, the equivalent resistor will be given by

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$$

Sample Problem 3

Find the equivalent resistor when a 4.0-Ω bulb and an 8.0-Ω bulb are connected in parallel.

Solution

$R_1 = 8.0\ \Omega$

$R_2 = 4.0\ \Omega$

$R_P = ?$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_P} = \frac{1}{8.0\ \Omega} + \frac{1}{4.0\ \Omega}$$

$$= \frac{1}{8.0\ \Omega} + \frac{2}{8.0\ \Omega}$$

$$\frac{1}{R_P} = \frac{3}{8.0\ \Omega}$$

$$R_P = \frac{8.0\ \Omega}{3}$$

$$R_P = 2.7\ \Omega$$

The equivalent resistor is 2.7 Ω.

Practice

Understanding Concepts

9. Find the equivalent resistance in each of these cases.
   (a) 16 Ω and 8.0 Ω connected in parallel
   (b) 22 Ω, 12 Ω, and 5.0 Ω connected in parallel
10. Calculate the equivalent resistance of two, three, four, and five 60-Ω bulbs in parallel. What is the simple relationship for the equivalent resistance of n equal resistances in parallel?

Answers

9. (a) 5.3 Ω
   (b) 3.0 Ω
10. 30 Ω; 20 Ω; 15 Ω; 12 Ω

Electric Circuit Analysis

With what you've learned about electricity you should be able to do a complete analysis of any simple series or parallel electric circuit containing resistances. However, because so many different electric circuits are possible, there is no standard approach to analyzing a circuit. The steps to take in each case will depend on the information you have about the circuit and what you want to find out.

Sample Problem 4

Find V_1, V_2, V_3, I_1, I_2, I_3, R_2 for the circuit in **Figure 6**.

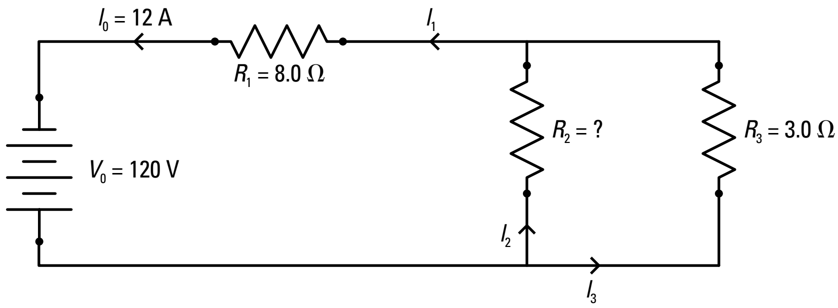

Figure 6

Solution

Using Kirchhoff's current law,

$$I_0 = I_1 = I_2 + I_3$$

Applying Ohm's law to the entire circuit

$$R_0 = \frac{V_0}{I_0}$$
$$= \frac{120\ \text{V}}{12\ \text{A}}$$
$$R_0 = 1.0 \times 10^1\ \Omega$$

If the parallel pair of resistors R_2 and R_3 is, for the moment, thought of as one single resistor, R_P, then R_1 and R_P are connected in series, and their total resistance is given by

$$R_t = R_1 + R_P$$
$$R_P = R_t - R_1$$
$$= 1.0 \times 10^1\ \Omega - 8.0\ \Omega$$
$$R_P = 2.0\ \Omega$$

Then, using the relationship for the equivalent resistance in parallel,

$$\frac{1}{R_P} = \frac{1}{R_2} + \frac{1}{R_3}$$
$$\frac{1}{R_2} = \frac{1}{R_P} - \frac{1}{R_3}$$
$$= \frac{1}{2.0\ \Omega} - \frac{1}{3.0\ \Omega}$$
$$= \frac{3}{6.0\ \Omega} - \frac{2}{6.0\ \Omega}$$
$$\frac{1}{R_2} = \frac{1}{6.0\ \Omega}$$
$$R_2 = 6.0\ \Omega$$

Using Ohm's law,

$$V_1 = I_1R_1$$
$$= (12\text{ A})(8.0\text{ V})$$
$$V_1 = 96\text{ V}$$

Then, applying Kirchhoff's voltage law around each of the two paths through the circuit,

$$V_0 = V_1 + V_2 \qquad\qquad V_0 = V_1 + V_3$$
$$V_2 = V_0 - V_1 \qquad\qquad V_3 = V_0 - V_1$$
$$= 120\text{ V} - 96\text{ V} \qquad\qquad = 120\text{ V} - 96\text{ V}$$
$$V_2 = 24\text{ V} \qquad\qquad V_3 = 24\text{ V}$$

Finally,

$$I_2 = \frac{V_2}{R_2} \qquad\qquad I_3 = \frac{V_3}{R_3}$$
$$= \frac{24\text{ V}}{6.0\ \Omega} \qquad\qquad = \frac{24\text{ V}}{3.0\ \Omega}$$
$$I_2 = 4.0\text{ A} \qquad\qquad I_3 = 8.0\text{ A}$$

As a check, $I_0 = I_2 + I_3 = 4.0\text{ A} + 8.0\text{ A} = 12\text{ A}$, as given.

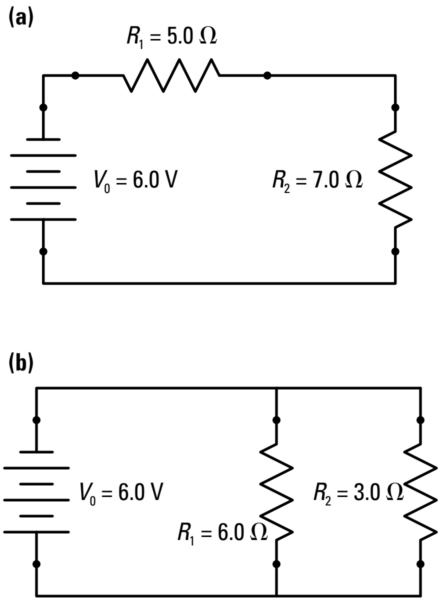

Figure 7
For question 11

Practice

Understanding Concepts

11. Complete each of the circuits in **Figure 7** by finding the unknown quantities indicated.
    (a) I_0, I_1, I_2, V_1, V_2
    (b) V_1, V_2, I_0, I_1, I_2

Answers

11. (a) $I_0 = 0.5$ A, $I_1 = 0.5$ A, $I_2 = 0.5$ A, $V_1 = 2.5$ V, $V_2 = 3.5$ V
    (b) $V_1 = 6.0$ V, $V_2 = 6.0$ V, $I_0 = 3.0$ A, $I_1 = 1.0$ A, $I_2 = 2.0$ A

SUMMARY Electric Resistance

- When charges pass through a material or device, they experience an opposition known as resistance.
- Ohm's law is represented by the equation $\frac{V}{I} = R$.
- The unit of electric resistance is called the ohm (Ω).
- If the number of resistors connected in series is n, the equivalent resistor is given by $R_S = R_1 + R_2 + ... + R_n$.
- If the number of resistors connected in parallel is n, the equivalent resistor is given by $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}$.

Section 12.6 Questions

Understanding Concepts

1. Find V_1, V_2, I_0, I_1, and R_2 for the circuit in **Figure 8**.
2. Find V_0, V_1, I_2, R_1, and R_2 for the circuit in **Figure 9**.

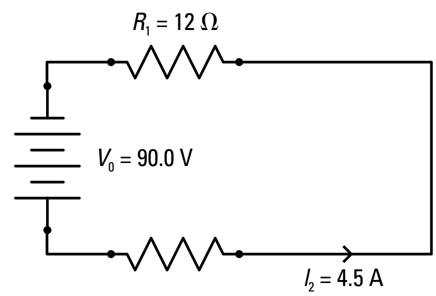

Figure 8
For question 1

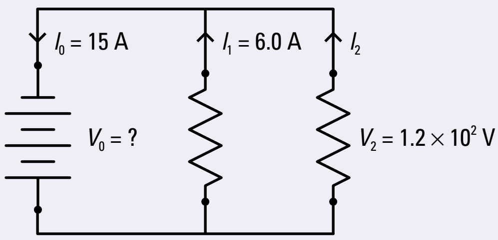

Figure 9

3. Find V_1, V_3, I_1, I_2, I_3, and R_3 for the circuit in **Figure 10**.

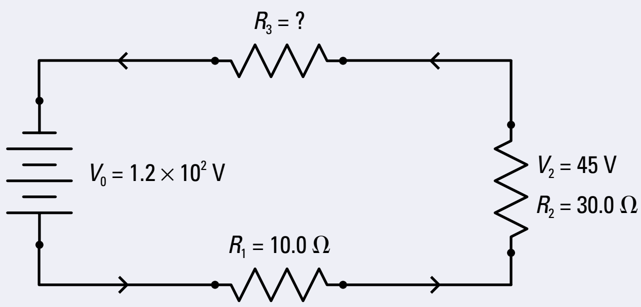

Figure 10

4. Find V_0, V_1, V_2, V_3, I_0, I_1, and I_2 for the circuit in **Figure 11**.

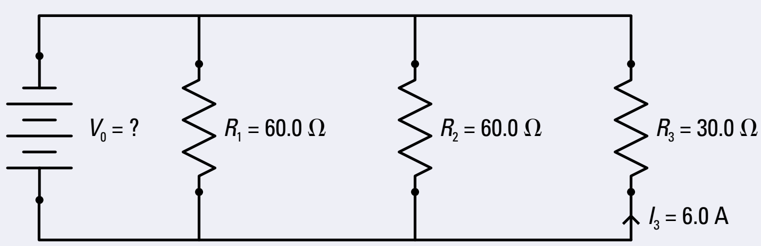

Figure 11

5. For each of the circuits in **Figure 12**, find the current through and the potential drop across each resistor.
6. What is the effect on the total resistance in a circuit when an extra resistor is added (a) in series and (b) in parallel?
7. Draw a graph of electric potential difference versus current for two different resistances, and indicate which has the greater resistance, and why.

(continued)

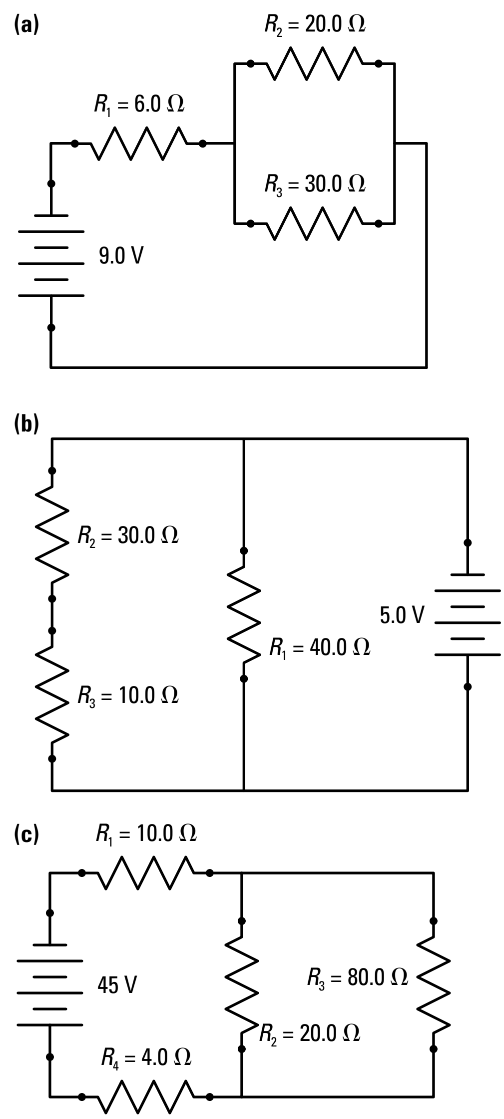

Figure 12
For question 5

Applying Inquiry Skills

8. A 12-V battery, an ammeter, a 5.0-A fuse (which will burn out if more than 5.0 A of current is in the circuit), and several 10.0-Ω lamps are used in an experiment to find the effect of connecting resistances in parallel (**Figure 13**).

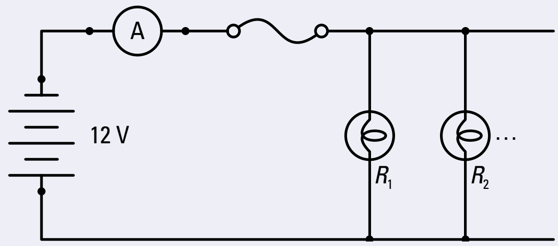

Figure 13

(a) Determine the total resistance and current when the number of lamps connected in parallel is 1, 2, 3, 4, 5, and 6.
(b) What is the maximum number of lamps that can be connected before the fuse becomes overloaded and burns out?
(c) Write at least one conclusion for the experiment.

9. Redraw the circuit shown in **Figure 14** and add the following:
(a) ammeters to find I_0, I_c, and I_e
(b) voltmeters to find the potential rise across the source and the potential drop across the two resistors in parallel
(c) a fuse at point a
(d) arrows indicating conventional electric current
(e) all positive and negative terminals

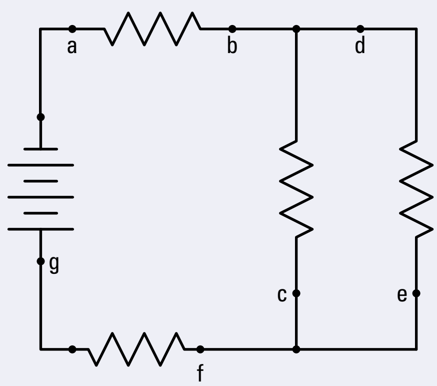

Figure 14

10. Parts of an electric circuit may heat up and start to smoke. What should you do if this occurs (a) in the classroom or (b) at home?

Making Connections

11. A dimmer switch is used in some household circuits to control the brightness of incandescent light bulbs. Describe how this type of switch operates. (A diagram will aid your explanation.)

12. The bulbs in an older string of Christmas tree lights are connected in series. They will all go out if one bulb is burnt out or removed. In a modern set, if one bulb burns out, the others stay lit. However, if a good bulb is removed or twisted, they all go out. Are modern sets connected in series or parallel?