It is difficult to study the properties of waves for sound, light, and radio because we cannot view the waves directly. However, if we use a ripple tank, not only can we view the waves directly, but we can create most conditions needed to demonstrate the properties of transverse waves in this two-dimension space. Investigation 9.1.1, in the Lab Activities section at the end of this chapter, provides you with an opportunity to study the properties of waves in a ripple tank in order to better understand and predict similar behaviours and relationships for other waves.

Transmission

A wave originating from a point source is circular, whereas a wave originating from a linear source is straight. We confine ourselves for the moment to waves from sources with a constant frequency. As a wave moves away from its constant-frequency source, the spacing between successive crests or successive troughs—the wavelength—remains the same provided the speed of the wave does not change. A continuous crest or trough is referred to as a wave front. To show the direction of travel, or transmission, of a wave front, an arrow is drawn at right angles to the wave front (Figure 1). This line is called a wave ray. Sometimes we refer to wave rays instead of wave fronts when describing the behaviour of a wave.

When the speed decreases, as it does in shallow water, the wavelength decreases (Figure 2), since wavelength is directly proportional to speed \( \lambda \propto v \). When the frequency of a source is increased, the distance between successive crests becomes smaller, since wavelength is inversely proportional to frequency \( \lambda \propto \frac{1}{f} \). Both proportionalities are consequences of the universal wave equation, \( v = f \lambda \). This equation holds for all types of waves—one-dimensional, two-dimensional, and three-dimensional.

The wave travelling in deep water has a speed \( v_1 = f_1 \lambda_1 \). Similarly, \( v_2 = f_2 \lambda_2 \) for the wave travelling in shallow water. In a ripple tank, the frequency of a water wave is determined by the wave generator and does not change when the speed changes. Thus \( f_1 = f_2 \).

If we divide the first equation by the second equation, we get

\[
\frac{v_1}{v_2} = \frac{f_1 \lambda_1}{f_2 \lambda_2}
\]

However, \( f_1 = f_2 \). Therefore,

\[
\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}
\]
A water wave has a wavelength of 2.0 cm in the deep section of a tank and 1.5 cm in the shallow section. If the speed of the wave in the shallow water is 12 cm/s, what is its speed in the deep water?

**Solution**

\[
\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}
\]

\[
v_1 = \left(\frac{\lambda_1}{\lambda_2}\right) v_2
\]

\[
= \left(\frac{2.0 \text{ cm}}{1.5 \text{ cm}}\right) 12 \text{ cm/s}
\]

\[
v_1 = 16 \text{ cm/s}
\]

The speed of the wave in deep water is 16 cm/s.

**Practice**

**Understanding Concepts**

1. The speed and the wavelength of a water wave in deep water are 18.0 cm/s and 2.0 cm, respectively. The speed in shallow water is 10.0 cm/s. Find the corresponding wavelength.

2. A wave travels 0.75 times as fast in shallow water as it does in deep water. Find the wavelength of the wave in deep water if its wavelength is 2.7 cm in shallow water.

3. In question 1, what are the respective frequencies in deep and shallow water?

**Answers**

1. 1.1 cm
2. 3.6 cm
3. 9.0 Hz; 9.0 Hz
angle of incidence ($\theta_i$) the angle between the incident wave front and the barrier, or the angle between the incident ray and the normal

angle of reflection ($\theta_r$) the angle between the reflected wave front and the barrier, or the angle between the reflected ray and the normal

refraction the bending effect on a wave’s direction that occurs when the wave enters a different medium at an angle

**Reflection from a Straight Barrier**

A straight wave front travels in the “wave ray” direction perpendicular to the wave front, but how will it behave when encountering obstacles? When a straight wave front runs into a straight reflective barrier, head on, it is reflected back along its original path (Figure 3). If a wave encounters a straight barrier obliquely (i.e., at an angle other than 90°), the wave front is likewise reflected obliquely. The angle formed by the incident wave front and the normal is equal to the angle formed by the reflected wave front and the normal. These angles are called the angle of incidence ($\theta_i$) and the angle of reflection ($\theta_r$), respectively (Figure 4). Reflection leaves wavelength, speed, and frequency unchanged.

---

**Figure 3**
A straight wave front meeting a straight barrier head on is reflected back along its original path.

**Figure 4**
When a wave encounters a straight barrier obliquely, rather than head on, the angle of incidence equals the angle of reflection.

---

**Refraction**

When a wave travels from deep water to shallow water in such a way that it meets the boundary between the two depths straight on, no change in direction occurs. On the other hand, if a wave meets the boundary at an angle, the direction of travel does change. This phenomenon is called refraction (Figure 5).
We usually use wave rays to describe refraction. The **normal** is a line drawn at right angles to a boundary at the point where an incident wave ray strikes the boundary. The angle formed by an incident wave ray and the normal is called the angle of incidence, $\theta_i$. The angle formed by the normal and the refracted wave ray is called the **angle of refraction**, $\theta_R$.

When a wave travels at an angle into a medium in which its speed decreases, the refracted wave ray is bent (refracted) toward the normal, as in **Figure 5(a)**. If the wave travels at an angle into a medium in which its speed increases, the refracted wave ray is bent away from the normal, as in **Figure 5(b)**.

**Figure 6** shows geometrically that $\theta_i$ is equal to the angle between the incident wave front and the normal and that $\theta_R$ is equal to the angle between the refracted wave front and the normal. In the ripple tank, it is easier to measure the angles between the wave rays and the boundary, that is, $\theta_i'$ and $\theta_R'$.

To analyze wave fronts refracted at a boundary, the angles of incidence and refraction can be determined using the equations $\sin \theta_i = \frac{\lambda_1}{xy}$ and $\sin \theta_R = \frac{\lambda_2}{xy}$, respectively (**Figure 7**).

![Figure 7](image-url)

The ratio of the sines gives

$$\frac{\sin \theta_i}{\sin \theta_R} = \frac{\left(\frac{\lambda_1}{xy}\right)}{\left(\frac{\lambda_2}{xy}\right)}$$

which reduces to

$$\frac{\sin \theta_i}{\sin \theta_R} = \frac{\lambda_1}{\lambda_2}$$

For a specific change in medium, the ratio $\frac{\lambda_1}{\lambda_2}$ has a constant value. Recall Snell’s law from optics, $\sin \theta_i \propto \sin \theta_R$. This equation can be converted to $\sin \theta_i = n \sin \theta_R$. The constant of proportionality ($n$) and the index of refraction ($n$) are one and the same thing.
Consequently, we can write
\[
\frac{\sin \theta_1}{\sin \theta_R} = n
\]

This relationship holds for waves of all types, including light, which we will see shortly. When light passes from a vacuum into a substance, \(n\) is called the **absolute index of refraction**. (See Table 1 for a list of absolute indexes of refraction.) The value for the absolute index of refraction is so close to the value from air to a substance that we rarely distinguish between them. In this text, when we refer to the index of refraction, we will be referring to the absolute index of refraction.

**Table 1** Approximate Absolute Indexes of Refraction for Various Substances*

<table>
<thead>
<tr>
<th>Substance</th>
<th>Absolute Refractive Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>vacuum</td>
<td>1.000 000</td>
</tr>
<tr>
<td>air</td>
<td>1.000 29</td>
</tr>
<tr>
<td>ice</td>
<td>1.31</td>
</tr>
<tr>
<td>water</td>
<td>1.333</td>
</tr>
<tr>
<td>ethyl alcohol</td>
<td>1.36</td>
</tr>
<tr>
<td>turpentine</td>
<td>1.472</td>
</tr>
<tr>
<td>glass</td>
<td>1.50</td>
</tr>
<tr>
<td>Plexiglas</td>
<td>1.51</td>
</tr>
<tr>
<td>crown glass</td>
<td>1.52</td>
</tr>
<tr>
<td>polystyrene</td>
<td>1.59</td>
</tr>
<tr>
<td>carbon disulphide</td>
<td>1.628</td>
</tr>
<tr>
<td>flint glass</td>
<td>1.66</td>
</tr>
<tr>
<td>zircon</td>
<td>1.923</td>
</tr>
<tr>
<td>diamond</td>
<td>2.417</td>
</tr>
<tr>
<td>gallium phosphide</td>
<td>3.50</td>
</tr>
</tbody>
</table>

*Measured with a wavelength of 589 nm. Values may vary with physical conditions.

You will also recall that we derived a general equation for Snell’s law that applies to any two substances:
\[
n_1 \sin \theta_1 = n_2 \sin \theta_2
\]

where \(n_1\) is the index of refraction in the first medium, \(n_2\) is the index of refraction in the second medium, and \(\theta_1\) and \(\theta_2\) are angles in each respective medium.

For waves we found that \(\frac{\sin \theta_1}{\sin \theta_R} = \frac{\lambda_1}{\lambda_2}\), which we can generalize to \(\frac{\sin \theta_1}{\sin \theta_2} = \frac{\lambda_1}{\lambda_2}\). But from the universal wave equation, \(v = f\lambda\), we can show that \(\frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}\) since \(f\) is constant. Therefore, we can write
\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}
\]

The following sample problems will illustrate the application of these relationships in both in the ripple tank and for light.
A 5.0 Hz water wave, travelling at 31 cm/s in deep water, enters shallow water. The angle between the incident wave front in the deep water and the boundary between the deep and shallow regions is 50°. The speed of the wave in the shallow water is 27 cm/s. Find
(a) the angle of refraction in the shallow water
(b) the wavelength in shallow water

Solution
(a) \( f = 5.0 \text{ Hz} \) \( \theta_1 = 50.0° \)
\( v_1 = 31 \text{ cm/s} \) \( \theta_2 = ? \)
\( v_2 = 27 \text{ cm/s} \)
\[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2}
\]
\[
\sin \theta_2 = \left( \frac{v_2}{v_1} \right) \sin \theta_1
\]
\[
\sin \theta_2 = \left( \frac{27 \text{ cm/s}}{31 \text{ cm/s}} \right) \sin 50.0°
\]
\[
\theta_2 = 41.9, \text{ or } 42°
\]
The angle of refraction is 42°.

(b) \( \lambda_2 = \frac{v_2}{f_2} \) but \( f_2 = f_1 = 5.0 \text{ Hz} \)
\[
\lambda_2 = \frac{27 \text{ cm/s}}{5.0 \text{ Hz}}
\]
\( \lambda_2 = 5.4 \text{ cm} \)
The wavelength in shallow water is 5.4 cm.

For a light ray travelling from glass into water, find
(a) the angle of refraction in water, if the angle of incidence in glass is 30.0°
(b) the speed of light in water

Solution
From Table 1,
\( n_g = n_1 = 1.50 \) \( \theta_g = \theta_1 = 30.0° \)
\( n_w = n_2 = 1.333 \) \( \theta_w = \theta_2 = ? \)
(a) \[
\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}
\]
\[
\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 = \frac{n_1}{n_2} \sin 30.0°
\]
\[
\theta_2 = \frac{1.50 \sin 30.0°}{1.333}
\]
\( \theta_2 = 34.3° \)
The angle of refraction in water is 34.3°.
At higher angles of incidence, there is reflection as well as refraction. You can see such partial reflection–partial refraction on the right.

**total internal reflection** the reflection of light in an optically denser medium; it occurs when the angle of incidence in the denser medium is greater than a certain critical angle.
We have remarked that the frequency of a wave does not in general change when its speed changes. Since \( \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \), you might expect that the index of refraction and the amount of bending would not change for waves of different frequencies, provided the medium remains the same (e.g., water of the same depth in both cases).

Figure 11, however, shows that indexes of refraction do, in general, depend on wavelength. In Figure 11(a), the low-frequency (long-wavelength) waves are refracted, as indicated by a rod placed on the screen below the transparent ripple tank. The rod is exactly parallel to the refracted wave fronts. In Figure 11(b), the frequency has been increased (the wavelength decreased), with the rod left in the same position. The rod is no longer parallel to the refracted wave fronts. It appears that the amount of bending, and hence the index of refraction, is affected slightly by the frequency of a wave. We can conclude that, since the index of refraction represents a ratio of speeds in two media, the speed of the waves in at least one of those media must depend on their frequency. Such a medium, in which the speed of the waves depends on the frequency, is called a dispersive medium.

Figure 10
(a) Partial refraction–partial reflection
(b) At the critical angle
(c) Total internal reflection

Figure 11
(a) The refraction of straight waves, with a rod marker placed parallel to the refracted wave fronts.
(b) The refracted wave fronts of the higher frequency waves are no longer parallel to the marker.
We stated previously that the speed of waves depends only on the medium. This statement now proves to be an idealization. Nevertheless, the idealization is a good approximation of the actual behaviour of waves, since the dispersion of a wave is the result of minute changes in its speed. For many applications, it is acceptable to make the assumption that frequency does not affect the speed of waves.

**Summary Waves in Two Dimensions**

- The wavelength of a periodic wave is directly proportional to its speed.
- The frequency of a periodic wave is determined by the source and does not change as the wave moves through different media or encounters reflective barriers.
- All periodic waves obey the universal wave equation, \( v = f \lambda \).
- The index of refraction for a pair of media is the ratio of the speeds or the ratio of the wavelengths in the two media \( \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \).
- Snell’s law \( n = \frac{\sin \theta_i}{\sin \theta_R} \) holds for waves and for light.
- When a wave passes from one medium to another, the wavelength changes and partial reflection–partial refraction can occur.

### Section 9.1 Questions

**Understanding Concepts**

1. Straight wave fronts in the deep region of a ripple tank have a speed of 24 cm/s and a frequency of 4.0 Hz. The angle between the wave fronts and the straight boundary of the deep region is 40°. The wave speed in the shallow region beyond the boundary is 15 cm/s. Calculate
   (a) the angle the refracted wave front makes with the boundary
   (b) the wavelength in the shallow water

2. The following observations are made when a straight periodic wave crosses a boundary between deep and shallow water: 10 wave fronts cross the boundary every 5.0 s, and the distance across 3 wave fronts is 24.0 cm in deep water and 18.0 cm in shallow water.
   (a) Calculate the speed of the wave in deep water and in shallow water.
   (b) Calculate the refractive index.

3. Straight wave fronts with a frequency of 5.0 Hz, travelling at 30 cm/s in deep water, move into shallow water. The angle between the incident wave front in the deep water and the straight boundary between deep and shallow water is 50°. The speed of the wave in the shallow water is 27 cm/s.
   (a) Calculate the angle of refraction in the shallow water.
   (b) Calculate the index of refraction.
   (c) Calculate the wavelength in the shallow water.

4. Straight wave fronts in the deep end of a ripple tank have a wavelength of 2.0 cm and a frequency of 11 Hz. The wave fronts strike the boundary of the shallow section of the tank at an angle of 60° and are refracted at an angle of 30° to the boundary. Calculate the speed of the wave in the deep water and in the shallow water.

5. The speed of a sound wave in cold air (−20°C) is 320 m/s; in warm air (37°C), the speed is 354 m/s. If the wave front in cold air is nearly linear, find \( \theta_R \) in the warm air if \( \theta_i = 30° \).

6. A straight boundary separates two bodies of rock. Longitudinal earthquake waves, travelling through the first body at 7.75 km/s, meet the boundary at an angle of incidence of 20.0°. The wave speed in the second body is 7.72 km/s. Calculate the angle of refraction.

7. Under what conditions do wave rays in water and light rays exhibit total internal reflection?

8. Light travels from air into a certain transparent material of refractive index 1.30. The angle of refraction is 45°. What is the angle of incidence?

9. A ray of light passes from water, with index of refraction 1.33, into carbon disulphide, with index of refraction 1.63. The angle of incidence is 30.0°. Calculate the angle of refraction.