10.3 Diffraction Gratings

diffraction grating device whose surface is ruled with close, equally spaced, parallel lines for the purpose of resolving light into spectra; transmission gratings are transparent; reflection gratings are mirrored

A **diffraction grating**, a device used for wave analysis, has a large number of equally spaced parallel slits, which act as individual line sources of light. The wave analysis of the pattern produced by these openings resembles the analysis we considered for the double slit. The waves passing through the slits interfere constructively on the viewing screen when $\sin\theta_m = \frac{m\lambda}{d}$, where $m = 0, 1, 2, 3 \ldots$ is the order of the bright line or maximum. This condition for constructive interference is derivable from the path difference $\Delta l = d \sin\theta$ between successive pairs of slits in the grating (**Figure 1**).

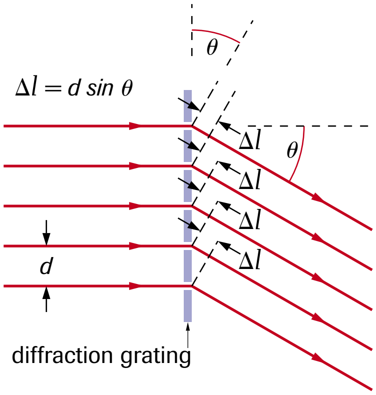


Figure 1
Light rays passing through a diffraction grating

The double-slit and the multi-slit interference patterns differ, however, in important ways. First, since there are more slits in the grating, the multi-slit source delivers more light energy, yielding a brighter interference pattern. Second, the bright maxima are much sharper and narrower when produced by a diffraction grating. Third, since the slits in a diffraction grating are usually closer together, the separation between successive maxima is greater than in the typical double-slit setup, and the resolution is enhanced. For these reasons, the diffraction grating is a very precise device for measuring the wavelength of light.

There are two common types of diffraction grating: transmission and reflection. To produce a grating of either kind, fine lines, to a density in some cases exceeding 10 000/cm, can be ruled on a piece of glass with a diamond tip. In a transmission grating, the spaces between the lines transmit the light, the lines themselves being opaque. When the lined glass is used as a reflection grating, light falling on it is reflected only in the untouched segments. This reflected light effectively comes from a series of equally spaced sources that provide the diffraction grating interference pattern reflected onto the screen. Lined gratings on shiny metal can also be used as a reflection diffraction grating (**Figure 2**).

DID YOU KNOW?

Other Types of Gratings

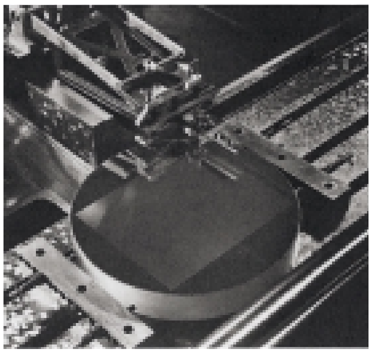
Another type of grating, called a *replica grating*, is made by pouring molten plastic over a master grating. When the plastic solidifies, it is peeled off the master and attached to glass or stiff plastic for support. The more modern holographic grating uses extremely narrow spacing to form an interference pattern on the photographic film.

TRY THIS activity — Grated Rulers

Shine a laser pointer, at a small angle of incidence, onto a metal ruler with etched markings. Direct the reflected light onto a screen or white wall. What do you see? What causes this effect?

Do not let direct laser beams or reflected beams go straight into anyone's eyes.

(a)



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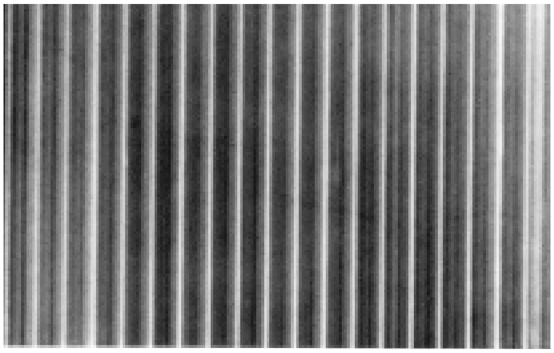

Figure 2
(a) A master grating being ruled by a diamond tip scribe
(b) A microscopic view of a ruled grating

SAMPLE problem

At what angle will 638-nm light produce a second-order maximum when passing through a grating of 900 lines/cm?

Solution

$\lambda = 638\ \text{nm} = 6.38 \times 10^{-7}\ \text{m}$

$d = \dfrac{1}{900\ \text{lines/cm}} = 1.11 \times 10^{-3}\ \text{cm} = 1.11 \times 10^{-5}\ \text{m}$

$m = 2$

$\theta = ?$

$$\sin \theta_m = \frac{m\lambda}{d} \quad \text{(for bright maxima)}$$

$$\sin \theta_2 = \frac{2(6.38 \times 10^{-7}\ \text{m})}{1.11 \times 10^{-5}\ \text{m}}$$

$$\sin \theta_2 = 1.15 \times 10^{-1}$$

$$\theta_2 = 6.60°$$

The angle to the second maximum is 6.60°.

LEARNING TIP

Exact Quantities
The quantity 900 lines/cm is a counted quantity, not a measured quantity, and thus is considered exact.

Practice

Understanding Concepts

1. A 4000-line/cm grating, illuminated with a monochromatic source, produces a second-order bright fringe at an angle of 23.0°. Calculate the wavelength of the light.
2. A diffraction grating with slits 1.00×10^{-5} m apart is illuminated by monochromatic light with a wavelength of 6.00×10^{2} nm. Calculate the angle of the third-order maximum.
3. A diffraction grating produces a third-order maximum, at an angle of 22°, for red light (694.3 nm). Determine the spacing of the lines in centimetres.
4. Calculate the highest spectral order visible when a 6200-line/cm grating is illuminated with 633-nm laser light.

Answers

1. 4.88×10^{-7} m
2. 10.4°
3. 7.4×10^{-4} cm
4. $n = 2$

Crossed Gratings and the Spectroscope

If you look through a grating with the gratings perpendicular to the light source, you see a horizontal diffraction pattern. When you rotate the grating through 90°, you see a vertical pattern. If you look at a point source of light through two such gratings, you will get a pattern such as that seen in **Figure 3**, called a *crossed-grating* diffraction pattern.

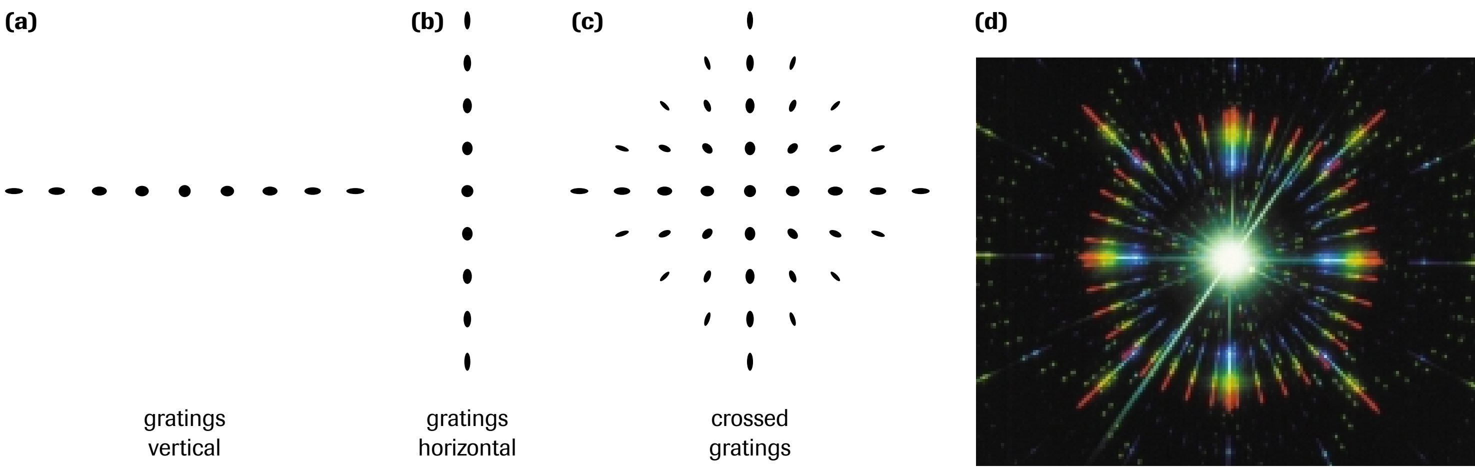

Figure 3
(a) A vertical grating produces a horizontal pattern.
(b) When the grating is horizontal, the pattern is vertical.
(c) Cross the gratings and the pattern is also crossed.
(d) A point source of light is viewed through crossed gratings.

DID YOU KNOW?

Dispersion with a Diffraction Grating

When a prism disperses white light, the greatest deflection is in violet light, the least in red light. With a diffraction grating, it is red light that undergoes the greatest deflection, violet light the least.

TRY THIS *activity* — Crossed Gratings

Take an ordinary handkerchief into a darkened room. Pull it taut and look at a distant point source of light through the fabric. (The point source should *not* be a laser beam.) In what way does the resulting diffraction pattern resemble **Figure 3**? Pull the cloth diagonally so that the mesh of the fabric is at an angle. How does the pattern change? Repeat with a coarser fabric, such as a tea towel, and with a finer fabric, such as a scrap of panty hose. What changes occur? Why? On a dark night, look at a bright star through an umbrella. Describe what you see.

If the light striking the grating is not monochromatic but a mixture of wavelengths, each wavelength produces a pattern of bright maxima at different locations on the viewing screen. This is what occurs when white light is directed through a diffraction grating. The central maximum will be a sharp white peak, but for each of the spectral colours, the maxima occur at different positions on the screen, giving the effect of a spectrum. The display is the same as when white light passes through a prism onto a viewing screen, except that the spectral pattern produced by a diffraction grating is much more widely spread out and easier to observe, making it an excellent device for spectral analysis. The instrument that performs this analysis is called a **spectroscope**.

spectroscope an instrument that uses a diffraction grating to visually observe spectra

In a spectroscope, light from a source is first directed through a collimator, a system of mirrors or lenses that makes the rays from the source essentially parallel. This parallel light passes through a diffraction grating. We view the resulting interference pattern with a small telescope. Since we can measure the angle θ quite accurately, we can determine the wavelength of the light to a high degree of accuracy (**Figure 4**).

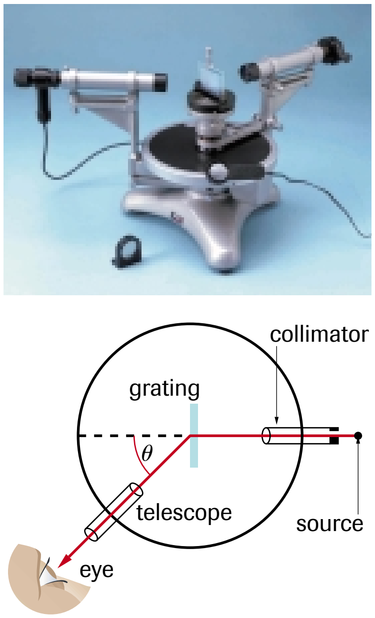

Figure 4
A spectroscope

TRY THIS *activity* — Using a Grating Spectroscope

View several gas discharge sources of light through a spectroscope, for example, hydrogen and neon, or use a fluorescent light (**Figure 5**). If you have the equipment, measure the angles for specific spectral lines for one of your sources, for example, hydrogen. Calculate their wavelengths, as directed by your teacher.

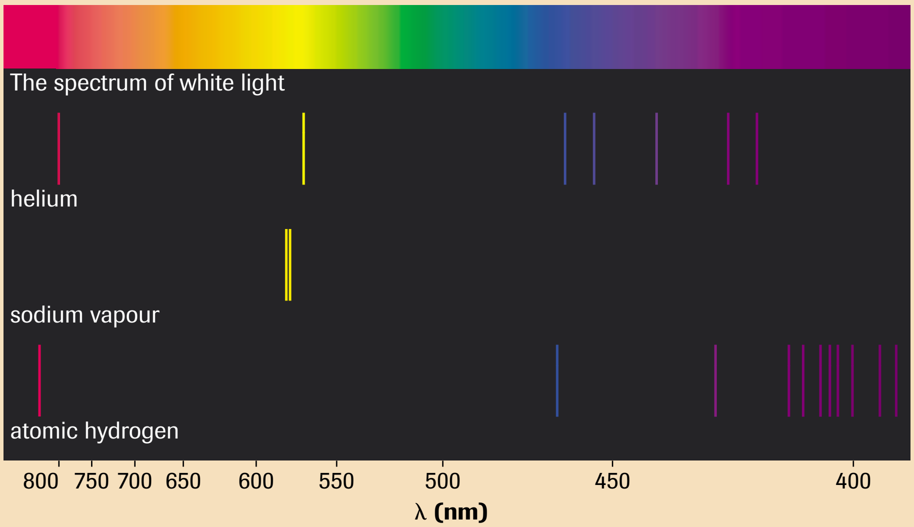

Figure 5
Spectra of white light, helium, sodium vapour, and hydrogen as produced by a spectroscope.

SUMMARY *Diffraction Gratings*

- The surface of a diffraction grating consists of a large number of closely spaced, parallel slits.
- Diffraction gratings deliver brighter interference patterns than typical double-slit setups, with maxima that are narrower and more widely separated.
- Diffraction gratings are governed by the relationship $\sin \theta_m = \frac{m\lambda}{d}$, where d is the distance between adjacent gratings, and m is the order of the maxima.

Section 10.3 Questions

Understanding Concepts

1. CDs reflect the colours of the rainbow when viewed under white light. What type of surface must be on the CD (**Figure 6**)? Explain your reasoning.

Figure 6

2. At what angle will 6.50×10^2-nm light produce a second-order maximum when falling on a grating with slits 1.15×10^{-3} cm apart?
3. Light directed at a 10 000-line/cm grating produces three bright lines in the first-order spectrum, at angles of 31.2°, 36.4°, and 47.5°. Calculate the wavelengths of the spectral lines in nanometres. What are the colours?
4. The "Balmer α" and "Balmer δ" lines in the visible atomic-hydrogen spectrum have wavelengths of 6.56×10^2 nm and 4.10×10^2 nm, respectively. What will be the angular separation, in degrees, of their first-order maxima, if these wavelengths fall on a grating with 6600 lines/cm?
5. A diffraction grating gives a first-order maximum at an angle of 25.0° for 4.70×10^2-nm violet light. Calculate the number of lines per centimetre in the grating.

Applying Inquiry Skills

6. Fog a piece of glass with your breath and look at a point source of light in a darkened room. What do you see? Why? Where in the sky have you seen a similar effect? Explain your answer.

Making Connections

7. A certain spectroscope, while in vacuum, diffracts 5.00×10^2-nm light at an angle of 20° in the first-order spectrum. The same spectroscope is now taken to a large, distant planet with a dense atmosphere. The same light is now diffracted by 18°. Determine the index of refraction of the planet's atmosphere.