You have probably noticed the swirling colours of the spectrum that result when gasoline or oil is spilled on water. And you have also seen the colours of the spectrum shining on a soap bubble. These effects are produced through optical interference, when light is reflected by or transmitted through a thin film.

**TRY THIS activity**

**Soap Bubbles**

Pour a small amount of bubble solution onto a clean plastic tray, such as a cafeteria tray. Use a straw to blow a bubble at least 20 cm in diameter. Direct a bright light onto the domed soap film. Note bright areas of constructive interference (different colours) and dark areas of destructive interference. These bright and dark areas correspond to variations in the thickness of the film and the movement of the water in the film.

Consider a horizontal film like a soap bubble that is extremely thin, compared to the wavelength of monochromatic light being directed at it from above, in air. When the light rays strike the upper surface of the film, some of the light is reflected, and some is refracted. Similar behaviour occurs at the lower surface. As a result, two rays are reflected to the eye of an observer: one (ray 1) from the top surface and another (ray 2) from the bottom (Figure 1). These two light rays travel along different paths. Whether they interfere constructively or destructively depends on their phase difference when they reach the eye.

Recall that when waves pass into a slower medium, the partially reflected waves are inverted (so that a positive pulse is reflected as a negative). When the transition is from a slow medium to a fast medium, reflected waves are not inverted. Transmitted waves are never inverted. Since both rays originate from the same source, they are initially in phase. Ray 1 will be inverted when it is reflected, whereas ray 2 will not. Because the film is very thin \( t \ll \lambda \), the extra distance travelled by ray 2 is negligible, and the two rays, being 180° out of phase, interfere destructively (Figure 2(a)). For this reason, a dark area occurs at the top of a vertical soap film, where the film is very thin.
Path difference approaches zero

(path difference is \( \frac{\lambda}{2} \))

(path difference is \( \lambda \))

Let us now consider what happens if the soap film is a little thicker (Figure 2(b)). In this case, ray 2 will have an appreciable path difference in comparison with ray 1. If the thickness \( t \) of the film is \( \frac{\lambda}{4} \), the path difference is \( 2 \times \frac{\lambda}{4} \), or \( \frac{\lambda}{2} \), for nearly normal paths, yielding a 180° phase delay. The two rays, initially out of phase because of reflection, are now back in phase again. Constructive interference occurs, and a bright area is observed.

When the thickness of the film is \( \frac{\lambda}{2} \) and the path difference is \( \lambda \) (Figure 2(c)), the two reflected rays are again out of phase and there is destructive interference.

There will thus be dark areas for reflection when the thickness of the film is \( 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots \). For the same reason, bright areas will occur when the thickness is \( \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots \). In these conditions for destructive and constructive interference, \( \lambda \) is the wavelength of light in the film, which would be less than the wavelength in air by a factor of \( n \), the index of refraction.

**TRY THIS** activity  
**Interference in Soap Bubbles**

Cover a clear showcase lamp with a red filter. Dip a wire loop into a soap solution. View the soap film by reflected red light, holding the loop in a vertical position for at least one minute. Remove the filter from the light so that white light strikes the soap film. Arrange the white light and the soap film so that the light passes through the film. Compare the respective patterns for transmitted and reflected light.
The effects of gravity on a vertical soap film cause the film to be wedge-shaped, thin at the top and thick at the bottom, with the thickness changing in a reasonably uniform way. The thickness changes uniformly, producing successive horizontal segments of dark and bright reflections, similar in appearance to the double-slit pattern under monochromatic light (Figure 3).

When viewed under white light, the soap film produces a different effect. Since the spectral colours have different wavelengths, the thickness of the film required to produce constructive interference is different for each colour. For example, a thickness of \( \frac{\lambda}{4} \) for red light is greater than the corresponding \( \frac{\lambda}{4} \) thickness for blue light, since red light has a longer wavelength in the film than blue light. Blue light and red light will therefore reflect constructively at different film thicknesses. When white light is directed at the film, each colour of the spectrum is reflected constructively from its own particular film thickness, and the spectral colours are observed.

Interference also occurs when light is transmitted through a thin film. We will use the soap film as an example. When the thickness \( t \) of the film is essentially zero (\( t \ll \lambda \)), the transmitted light can be considered to consist of two rays: ray 1, transmitted without any phase change, and ray 2, reflected twice internally and likewise transmitted without a phase change. (Recall that for reflection from a slow medium to a fast medium there is no phase change.) Since the path difference is negligible, the two rays emerge in phase, yielding a bright area of constructive interference (Figure 4(a)). This is the opposite of the result for light reflected from the same thin film. Dark areas of destructive interference for transmission occur when the thickness is \( \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots \) (Figure 4(b)). Bright transmission areas appear when the thickness is \( 0, \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots \) (Figure 4(c)). (Remember that the path difference is twice the film thickness.) In all cases, \( \lambda \) is the wavelength of the light in the film material, which will be less than the wavelength of light in air by a factor of \( n \), the index of refraction.
In summer months, the amount of solar energy entering a house should be minimized. Window glass is made energy-efficient by applying a coating to maximize reflected light. Light in the midrange of the visible spectrum (at 568 nm) travels into energy-efficient window glass, as in Figure 5. What thickness of the added coating is needed to maximize reflected light and thus minimize transmitted light?

**Solution**

Reflection occurs both at the air–coating interface and at the coating–glass interface. In both cases, the reflected light is 180° out of phase with the incident light, since both reflections occur at a fast-to-slow boundary. The two reflected rays would therefore be in phase if there were zero path difference. To produce constructive interference the path difference must be \( \frac{\lambda}{4} \). In other words, the coating thickness \( t \) must be \( \frac{\lambda}{4} \), where \( \lambda \) is the wavelength of the light in the coating.

\[
\begin{align*}
  n_{\text{coating}} &= 1.4 \\
  t &= ? \\
  n_{\text{coating}} &= \frac{\lambda_{\text{air}}}{\lambda_{\text{coating}}} \\
  \lambda_{\text{coating}} &= \frac{\lambda_{\text{air}}}{n_{\text{coating}}} \\
  &= \frac{568 \text{ nm}}{1.4} \\
  &= 406 \text{ nm} \\
  t &= \frac{\lambda_{\text{coating}}}{4} \\
  &= \frac{406 \text{ nm}}{4} \\
  &= 101 \text{ nm}, \ or \ 1.0 \times 10^{-7} \text{ m}
\end{align*}
\]

The required thickness for the coating is \( 1.0 \times 10^{-7} \text{ m} \).
Mathematical Analysis of Interference in an Air Wedge

The soap film wedge, when held vertically and illuminated, produced bands of interference that were irregular. If an air wedge is created between two uniform pieces of glass and illuminated, a measurable pattern of constructive and destructive interference also results (Figure 6). This air wedge can be used to find the wavelength of the incident light. More importantly, it can be used to measure the size of very small objects.

The spacing of successive dark fringes in the reflection interference pattern of an air wedge may be calculated as follows (Figure 7):

Consider points G and F, where the glass-to-glass distances across the air wedge are \( \frac{\lambda}{2} \) and \( \lambda \), respectively. At point G, there is both transmission and reflection.

---

**Answers**

1. 0, 242 nm; 485 nm
2. 193 nm
3. 5.50 \( \times \) 10² nm
4. 233 nm

---

**Practise**

**Understanding Concepts**

1. What are the three smallest thicknesses of a soap bubble capable of producing reflective destructive interference for light with a wavelength of 645 nm in air? (Assume that the index of refraction of soapy water is the same as that of pure water, 1.33.)

2. A thin layer of glass (\( n = 1.50 \)) floats on a transparent liquid (\( n = 1.35 \)). The glass is illuminated from above by light with a wavelength, in air, of 5.80 \( \times \) 10² nm. Calculate the minimum thickness of the glass, in nanometres, other than zero, capable of producing destructive interference in the reflected light. Draw a diagram as part of your solution.

3. A coating, 177.4 nm thick, is applied to a lens to minimize reflections. The respective indexes of refraction of the coating and of the lens material are 1.55 and 1.48. What wavelength in air is minimally reflected for normal incidence in the smallest thickness? Draw a diagram as part of your solution.

4. A transparent oil (\( n = 1.29 \)) spills onto the surface of water (\( n = 1.33 \)), producing a maximum of reflection with normally incident orange light, with a wavelength of 6.00 \( \times \) 10⁻² m in air. Assuming the maximum occurs in the first order, determine the thickness of the oil slick. Draw a diagram as part of your solution.

---

**Figure 6**

Interference in an air wedge illuminated by mercury light
Since the light is going from a slow to a fast medium, there is no phase change in ray 1. At point B, reflection is from a fast to a slow medium, yielding a phase change. But by the time ray 2 lines up with ray 1, it has travelled two widths of GB, or one wavelength, farther, keeping the phase change intact. The path difference between ray 1 and ray 2 is one wavelength, and ray 1 and 2 interfere destructively, since they are 180° out of phase. Similarly, destructive interference occurs at point F, because ray 1’ and ray 2’ are shifted 180° out of phase upon reflection and remain out of phase because ray 2’ has had to travel two wavelengths farther.

By similar triangles, we have, for the first dark fringe, \( \triangle ABG \cong \triangle ADE \).

Therefore,

\[
\frac{x_1}{L} = \frac{\lambda}{2t}
\]

\[
x_1 = \frac{L\lambda}{2t}
\]

Similarly, for the second dark fringe, \( \triangle ACF \cong \triangle ADE \).

\[
\frac{x_2}{L} = \frac{\lambda}{t}
\]

\[
x_2 = \frac{L\lambda}{t}
\]

Since \( \Delta x = x_2 - x_1 \)

\[
= \frac{L\lambda}{t} - \frac{L\lambda}{2t}
\]

\[
\Delta x = \frac{L\lambda}{t} \left(1 - \frac{1}{2}\right), \text{ or}
\]

\[
\Delta x = L\left(\frac{\lambda}{2t}\right)
\]

where \( \Delta x \) is the distance between dark fringes, \( L \) is the length of the air wedge, \( t \) is the thickness of the base of the wedge, and \( \lambda \) is the wavelength of the light in the wedge.

Try applying the theory by performing Investigation 10.4.1 in the Lab Activities section at the end of this chapter, where you can measure the width of a strand of hair.

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**INVESTIGATION 10.4.1**

**Interference in Thin Films and Air Wedges (p. 542)**

How can you precisely measure the widths of tiny objects, objects too tiny to use conventional measuring instruments? This investigation provides an opportunity to measure the wavelength of light as well as the width of a very thin object: a single strand of your own hair.
(a) An air wedge between two microscope slides, 11.0 cm long and separated at one end by a paper of thickness 0.091 mm, is illuminated with red light of wavelength 663 nm. What is the spacing of the dark fringes in the interference pattern reflected from the air wedge?

(b) How would the spacing change if the wedge were filled with water ($n = 1.33$)?

**Solution**

$L = 11.0 \text{ cm}$

$t = 0.091 \text{ mm} = 9.1 \times 10^{-3} \text{ cm}$

$\lambda = 663 \text{ nm} = 6.63 \times 10^{-5} \text{ cm}$

$\Delta x = ?$

(a) In air:

$$\Delta x = L \left( \frac{\lambda}{2t} \right)$$

$$= 11.0 \text{ cm} \left( \frac{6.63 \times 10^{-5} \text{ cm}}{2(9.1 \times 10^{-3} \text{ cm})} \right)$$

$$\Delta x = 4.0 \times 10^{-2} \text{ cm}$$

The spacing between the dark fringes in air is $4.0 \times 10^{-2} \text{ cm}$.

(b) If the air were replaced by water:

$$\frac{n_w}{n_a} = \frac{\lambda_{air}}{\lambda_{water}}$$

$$\lambda_{water} = \left( \frac{n_a}{n_w} \right) \lambda_{air}$$

$$= \frac{1.00}{1.33} \left( 6.63 \times 10^{-5} \text{ cm} \right)$$

$$\lambda_{water} = 4.98 \times 10^{-5} \text{ cm}$$

$$\Delta x = L \left( \frac{\lambda}{2t} \right)$$

$$= 11.0 \text{ cm} \left( \frac{4.98 \times 10^{-5} \text{ cm}}{2(9.1 \times 10^{-3} \text{ cm})} \right)$$

$$\Delta x = 3.0 \times 10^{-2} \text{ cm}$$

The spacing of the dark fringes in water would be $3.0 \times 10^{-2} \text{ cm}$.

**Practice**

**Understanding Concepts**

5. Two pieces of glass forming an air wedge 9.8 cm long are separated at one end by a piece of paper $1.92 \times 10^{-3} \text{ cm}$ thick. When the wedge is illuminated by monochromatic light, the distance between centres of the first and eighth successive dark bands is 1.23 cm. Calculate the wavelength of the light.

6. Light with a wavelength of $6.40 \times 10^2 \text{ nm}$ illuminates an air wedge 7.7 cm long, formed by separating two pieces of glass with a sheet of paper. The spacing between fringes is 0.19 cm. Calculate the thickness of the paper.

7. A piece of paper is placed at the end of an air wedge 4.0 cm long. Interference fringes appear when light of wavelength 639 nm is reflected from the wedge. A dark fringe occurs both at the vertex of the wedge and at its paper end, and 56 bright fringes appear between. Calculate the thickness of the paper.
Now that we have studied how the wave theory applies to thin films and the mathematics of air wedges, we can apply this knowledge in the next section to examine some real-world applications of thin films.

**SUMMARY**

### Interference in Thin Films

- For reflected light in thin films, destructive interference occurs when the thin film has a thickness of $0, \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots$, and constructive interference occurs at thicknesses of $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$, where $\lambda$ is the wavelength in the film.
- For transmitted light in thin films, destructive interference occurs at $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots$, and constructive interference occurs at $0, \frac{\lambda}{2}, \frac{3\lambda}{2}, \ldots$, where $\lambda$ is the wavelength in the film.
- Air wedges can be used to determine the thicknesses of very small objects through the relationship $\Delta x = L \left( \frac{\lambda}{2t} \right)$.

### Section 10.4 Questions

#### Understanding Concepts

1. When light is transmitted through a vertical soap film, the top of the film appears bright, not dark, as it is when viewed from the other side. Explain, using diagrams and the wave theory of light.

2. Explain why it is usually not possible to see interference effects in thick films.

3. What is the minimum thickness of an air layer between two flat glass surfaces (a) if the glass is to appear bright when $4.50 \times 10^2$ nm light is incident at $90^\circ$? (b) if the glass is to appear dark? Use a diagram to explain your reasoning in both cases.

4. A film of gasoline ($n = 1.40$) floats on water ($n = 1.33$). Yellow light, of wavelength $5.80 \times 10^2$ nm, shines on this film at an angle of $90^\circ$.
   (a) Determine the minimum nonzero thickness of the film, such that the film appears bright yellow from constructive interference.
   (b) What would your answer have been if the gasoline had been spread over glass ($n = 1.52$) rather than over water?

5. Two plane glass plates $10.0$ cm long, touching at one end, are separated at the other end by a strip of paper $1.5 \times 10^{-3}$ mm thick. When the plates are illuminated by monochromatic light, the average distance between consecutive dark fringes is $0.20$ cm. Calculate the wavelength of the light.

6. Two plane glass plates $12.0$ cm long, touching at one end, and separated at the other end by a strip of paper, are illuminated by light of wavelength $6.30 \times 10^{-5}$ cm. A count of the fringes gives an average of $8$ dark fringes per centimetre. Calculate the thickness of the paper.

#### Making Connections

7. Semiconductors, such as silicon, are used to fabricate solar cells. These cells are typically coated with a thin, transparent film to reduce reflection losses and increase the efficiency of the conversion of solar energy to electrical energy. Calculate the minimum thickness of the film required to produce the least reflection of light with a wavelength of $5.50 \times 10^{-5}$ m when a thin coating of silicon oxide ($n = 1.45$) is placed on silicon ($n = 3.50$).

8. The national standards for new home construction may include thin-film coatings, called “E-coatings,” on thermapane windows for energy conservation. Research this standard, and find out how E-coatings are used to reduce heat loss in winter and decrease heat gain in summer. Write a short report on your findings.