# **Frames of Reference and Relativity**

Newton's laws of motion work very well at low speeds, that is, low compared to the speed of light. Einstein's special theory of relativity analyzes the effects of motion at high speeds, that is speeds approaching the speed of light. Further, as the word "relative" implies, the results of a measurement can depend on the frame of reference with respect to which the measurement is made. Before investigating motion at high speeds, we first review the concepts of relative motion for objects at low speeds.

# **Frames of Reference**

In Unit 1, we saw that to describe and account for the motion of any object, we must adopt a frame of reference from which to view the motion—an arbitrary origin and set of axes from which to measure the changing position of a moving object. Most often we choose Earth as our frame of reference, assuming it to be stationary, and measure all positions of a moving object relative to some origin and set of axes fixed on Earth. It was, of course, in Earth's frame of reference that Newton's first law, the law of inertia, was discovered. Any frame of reference in which the law of inertia holds is called an **inertial frame of reference** (see Section 2.5); that is, if no net force acts on an object at rest, it remains at rest, or, if in motion, it continues to move in a straight line at a constant speed.

All of Newtonian physics, including gravitation theory and kinematics, holds as we make the transition from one inertial frame to another. Suppose (contrary to commonsense and highway safety laws) you are standing up in the back of a pickup truck, holding an apple. The truck is moving along a straight, level road at constant speed. If you drop the apple, you see it fall, relative to the truck body, straight down (**Figure 1(a)**). But to an observer at the side of the road, in what we will call the Earth frame of reference, the path of the apple is a curve (**Figure 1(b**)). How do the laws of Newtonian physics compare in the two frames?

In both cases, the force of gravity accelerates the apple straight down. The observed vertical trajectory of the apple in the frame of the truck is correctly predicted by classical



## inertial frame of reference a

frame of reference in which the law of inertia holds

## Figure 1

An apple is dropped in a pickup truck.

- (a) In the frame of reference of the truck, the apple falls straight down.
- (b) In the frame of reference of Earth, the apple follows a parabolic path.

kinematics, since the apple has an initial horizontal velocity of zero in that frame. The observed curved trajectory of the apple in the frame of Earth is again correctly predicted by classical kinematics, since the apple has a nonzero initial horizontal velocity, directed forward, in that frame. In other words, the laws of physics (Newton's laws and the equations of kinematics) are the same in both frames of reference, even though the paths are different. We can generalize to say that the Newtonian laws of physics are the same in all inertial frames of reference.

Our experiences of travel make us familiar with the behaviour of a **noninertial frame** of **reference**, or a frame accelerated with respect to an inertial frame. When a vehicle changes speed, or turns sharply while maintaining a constant speed, odd things appear to happen. Consider a ball at rest on the flat, smooth, level floor of a van moving in a straight line on a level road at a constant speed. As long as the van is an inertial frame of reference, Newton's first law applies, and the ball remains at rest (**Figure 2(a**)).

**noninertial frame of reference** frame of reference that is accelerating relative to an inertial frame



When the speed of the vehicle increases on a straight and level road, the ball accelerates toward the rear of the van **Figure 2(b)**, contrary to Newton's law of inertia. Similarly, if the van slows down, the ball begins to move forward (**c**). If the road curves sharply to the right, the ball begins to move to the left (**d**). In each case, however, when the motion is observed from the inertial frame of Earth, the ball is seen to obey the law of inertia to continue moving in a straight line with a constant speed.

So firm is our belief in Newton's laws that we would rather invent a "fictitious force" to explain these strange motions in noninertial frames than abandon our belief in Newton's laws. In the previous example, we would have to assume a fictitious force in a direction opposite to that of the van's acceleration in order to explain the motion of the ball in each case. In the case where the van is turning, we make up a fictitious force, commonly called "centrifugal force," just to make intuitive sense. This is familiar to everyone who has taken a ride at an amusement park.

It is clear that the analysis of motion in noninertial frames is complicated. Looking at the same motion from any inertial frame provides a much simpler analysis, consistent with Newton's laws.

This leads us to three important statements about relative motion and frames of reference:

• In an inertial frame of reference, an object with no net force acting on it remains at rest or moving in a straight line with a constant speed.



### Figure 2

- (a) The van moves with constant velocity, and the ball stays at rest relative to the vehicle.
- (b) The van accelerates and the ball rolls backward relative to the vehicle.
- (c) The van slows down, and the ball rolls forward relative to the vehicle.
- (d) The ball rolls to the left relative to the vehicle.

In all cases the ball remains in uniform motion relative to Earth.

- The laws of Newtonian mechanics are only valid in an inertial frame of reference.
- The laws of Newtonian mechanics apply equally in all inertial frames of reference; in other words, all inertial frames of reference are equivalent as far as adherence to the laws of mechanics is concerned.

One final point remains to be made in our review of Newtonian relative motion and frames of reference: there is no such thing as absolute velocity in Newtonian mechanics. Whether you drop a ball while in a vehicle moving with constant velocity east, or in a vehicle moving with a constant velocity west, or in a parked vehicle, the ball moves vertically in the frame of the vehicle. Thus you cannot use measurements of the motion of the ball to help you identify whether you are really moving. In general, for any two inertial frames moving with respect to each other, there is no physical meaning in the question, "Which of these two frames is really moving?"

## **Special Theory of Relativity**

In Chapter 3, we learned how to calculate, by vector addition, relative velocities in moving frames of reference. We have just stressed in our review that Newton's laws of motion apply equally in all inertial frames. We now recall from Chapter 3 that the motion itself has a different appearance, depending on the frame from which it is viewed. For example, if a ball is rolled forward at 10 m/s in a car moving at 30 m/s, its speed is 40 m/s in Earth's frame of reference. Conversely, if the ball is rolled backward at the same speed in the same car, its speed relative to Earth is 20 m/s. Clearly, the speed with which the ball is observed to move depends on the frame of reference of the observer.

At the turn of the twentieth century, many physicists wondered whether the same vector-addition rules applied to the motion of light. If light has a speed *c* in a frame of reference of Earth, then would light emitted in the forward direction from a source moving relative to Earth at  $\frac{1}{10}$  *c* have a measured speed of  $\frac{11}{10}$  *c*, measured by an observer in Earth's frame of reference? Would light emitted backward from the same source have a measured speed of  $\frac{9}{10}$  *c* in Earth's frame? In other words, would the speed of light, like the speed of a ball rolling in a vehicle, depend on the frame of reference from which it is observed?

The first hint that light was somehow different from other phenomena came in the latter half of the nineteenth century, when Maxwell described light as an electromagnetic wave travelling in a vacuum at  $3.00 \times 10^8$  m/s. Relative to what frame of reference would the speed of light have this value? Did the calculation presuppose some special, absolute frame?

Up to this time, physicists had always associated waves with a medium through which they travelled. It was natural, then, for them to assume that light must also travel through some kind of medium. Perhaps this medium was the absolute frame of reference in the universe and the speed Maxwell calculated for electromagnetic waves was relative to this frame. The supposed medium, called the **ether**, was thought to allow bodies to pass through it freely, to be of zero density, and to permeate all of space.

According to classical mechanics, the speed of light measured relative to any frame of reference moving through this ether should differ from  $3.00 \times 10^8$  m/s by the magnitude of the velocity with which the frame is moving. It was assumed that Earth must be such a moving frame, since Earth is a planet orbiting the Sun. A number of very clever and complicated experiments were designed to measure the speed of Earth through the ether. The most successful of these was performed in 1887 by two Americans, A.A. Michelson (1852–1931) and E.W. Morley (1838–1923). While the details of the

## LEARNING TIP

Shortcut Symbols

Since the speed of light is a constant, it is given the symbol c. Thus,  $c = 3.00 \times 10^8$  m/s, so  $\frac{1}{2}c$ , or 0.5c, is equal to  $\frac{3.00 \times 10^8 \text{ m/s}}{2}$  or  $1.50 \times 10^8$  m/s.

**ether** the hypothetical medium, regarded as not directly observable, through which electromagnetic radiation was thought to propagate Michelson–Morley experiment can be left to a further course in physics, a brief description of their method and results is necessary for our understanding of special relativity.

In essence, Michelson and Morley compared the relative speeds of light in two perpendicular directions relative to Earth's motion through the ether (**Figure 3**). If Earth were travelling in the ether-absolute frame of reference with velocity  $\vec{v}$ , then in a frame on Earth the ether would be travelling at velocity  $-\vec{v}$ , producing an "ether wind." Michelson and Morley expected to find a difference in the measured speed of light dependent on the orientation of their apparatus in the ether wind. Just as the velocity, relative to the shore, of a boat with an outboard motor of constant power varies when the boat is directed first back and forth along the line of the river, then back and forth cross-stream, so the speed of light should differ when it is moving on the one hand back and forth along the line of the wind, and on the other hand perpendicular to the line. To detect the expected small difference in speed, Michelson and Morley used an interferometer, which generates an interference pattern between two parts of a split beam of light.

**Figure 4(a)** shows the setup of the apparatus. (See Section 10.7 for the operation of an interferometer.) The entire apparatus could be rotated to change the positions of the mirrors.

Any small difference in the velocity of light along the two paths would be indicated by a change in the interference pattern as the apparatus rotated. If the apparatus is rotated 90°, the distance  $L_1$  is now perpendicular to the ether wind and the distance  $L_2$  is parallel to it (**Figure 4(b)**). Thus, the time taken to travel these distances should change as the apparatus is rotated. This should produce a phase change in the interference pattern.



The importance of the experiment lies in its failure to show what was expected. Michelson and Morley performed their experiment over and over at different points in Earth's orbit but continued to get a null result: there was absolutely no change in the interference pattern. The speed of light was the same whether it travelled back and forth in the direction of the ether wind or at right angles to it. The relative velocity of the ether with respect to Earth had no effect on the speed of light. In other words, *the ether does not exist*. This null result was one of the great puzzles of physics at the turn of the twentieth century.

Many explanations were offered for the failure of the interference pattern to change. In 1905, Albert Einstein (1879–1955), then working in Switzerland as a junior patent clerk,



#### **Figure 3**

At most points during its orbit, Earth will be moving relative to the ether.

### Figure 4

- (a) A simplified view of an interferometer place in the hypothetical ether wind.
- (b) The apparatus is rotated 90°.

proposed a revolutionary explanation in the form of the *special theory of relativity*. His theory rests on two postulates.

## **Special Theory of Relativity**

- 1. *The relativity principle:* all the laws of physics are valid in all inertial frames of reference.
- 2. The constancy of the speed of light: light travels through empty space with a speed of  $c = 3.00 \times 10^8$  m/s, relative to all inertial frames of reference.

The first postulate is an easy-to-accept extension of the idea of Newtonian relativity, mentioned earlier. Einstein proposed that not only Newtonian mechanics but *all* the laws of physics, including those governing electricity, magnetism, and optics, are the same in all inertial frames. The second is more difficult to reconcile in our minds because it contradicts our commonsense notions of relative motion. We would expect two observers, one moving toward a light source and the other moving away from it, to make two different determinations of the relative speed of light. According to Einstein, however, each would obtain the same result,  $c = 3.00 \times 10^8$  m/s. Clearly, our everyday experiences and common sense are of no help in dealing with motion at the speed of light.

By doing away with the notion of an absolute frame of reference, Einstein's theory solves the dilemma in Maxwell's equations: the speed of light predicted by Maxwell is not a speed in some special frame of reference; it is the speed in *any* inertial frame of reference.

We have seen that in Newtonian mechanics, while the laws of motion are the same in all inertial frames, the appearance of any one particular motion is liable to change from frame to frame. We shall see in the rest of this chapter that the position for Einstein is similar but more radical: the changes in the appearance of the world, as we move between inertial frames travelling at high speeds with respect to each other, are contrary to common sense.

Note that special relativity is a special case of the more general theory of relativity (not investigated in this text), published by Einstein in 1916. The general theory of relativity deals with gravitation and noninertial frames of reference.

The special and general theories of relativity and their many implications are now considered as much a part of physics as Newton's laws. The difference is this: to comprehend the many ramifications of the theories requires a great deal more mental flexibility and dexterity than was the case with Newtonian mechanics.

## Simultaneity

We begin our examination of the consequences Einstein drew from his two postulates by considering time. In Newtonian mechanics, there is a universal time scale, the same for all observers. This seems right. Surely, a sequence of events that one observer measures to last 2.0 s would also last 2.0 s to an observer moving with respect to the first observer. But it is not always so! According to Einstein, time interval measurements depend on the reference frame in which they are made.

**Simultaneity**, the occurrence of two or more events at the same time, is also a relative concept, and we will make it our starting point, before proceeding to the relativity of a time interval. We will use a thought experiment to show that events that are simultaneous in one inertial frame are not simultaneous in other frames.

An observer  $O_s$ , stationary in the inertial frame of Earth, is standing on a railway platform at the midway point between two lampposts,  $L_1$  and  $L_2$  (**Figure 5**). The lampposts are connected to the same circuit, ensuring that, at least from the viewpoint of an iner-

## DID YOU KNOW

#### **Invariance or Relativity?**

Einstein originally used the name "theory of invariance" and only later the theory of relativity. In a sense, invariance describes the theory better than the word "relativity."

## DID YOU KNOW 🚽

## Precise Value of the Speed of Light

The speed of light is large but not infinite: 2.997 924 58  $\times$  10<sup>8</sup> m/s. For the calculations in this text, three significant digits are sufficient in most cases. Thus, 3.00  $\times$  10<sup>8</sup> m/s is used for the speed of light.

**simultaneity** the occurrence of two or more events at the same time



## Figure 5

- (a) When the train is at rest, each observer sees the lamps flash simultaneously, since each observer is halfway between the lamps.
- (b) When the train is moving, each observer does not see the lamps flash simultaneously, since the light from L<sub>1</sub> takes longer to reach  $O_m$  than the light from L<sub>2</sub>, that is  $\Delta t_{L1} > \Delta t_{L2}$ .

tial frame anchored in the railway platform, the lamps come on at the same time when the switch is closed. To make the experiment easy to follow, we assume that the lamps do not stay on when the current is applied but flash and explode, spewing out soot and broken glass. The light from each flash travels out in all directions at the speed of light, *c*. Since  $O_s$  is located at the midpoint between the lampposts, the distances the beams of light travel are equal, causing the arrival of light from the one lamp to be simultaneous with the arrival of light from the other lamp. Adjacent to  $O_s$  is a second observer  $O_m$ , sitting in a train on a straight track next to the platform. We make two trials in our thought experiment: in the first, keeping the train at rest relative to Earth, and in the second, making the train move with speed *v* relative to Earth.

If the train is at rest,  $O_m$  finds the arrival of the light from the first lamp to be simultaneous with the arrival of the light from the second lamp (**Figure 5(a)**).  $O_m$  then performs measurements of the soot marks left by the exploding lamps on his train: he is halfway between the soot marks, and light always travels at the same speed, *c*. Therefore,  $O_m$  is forced to conclude that the lamps flashed simultaneously.  $O_s$  reaches the same conclusion for the same reasons.

We now perform the second trial in the thought experiment, letting the train move by  $O_s$  at a high speed relative to the inertial frame of Earth but keeping everything else as before. In the time interval it takes for the flash of light to travel to  $O_s$  from each lamppost,  $O_m$  will have moved a short distance to the right (**Figure 5(b)**). In this time interval  $O_m$  will receive the flash of light from  $L_2$  but not yet receive the flash of light from  $L_1$ .  $O_m$  thus sees the rear lamp flash a little later than the forward lamp. Having taken this observation,  $O_m$  now performs measurements, as in the first trial: he is halfway between the soot marks left on the train, and light always travels at the same speed, *c*. Therefore,  $O_m$  is forced to conclude that the two lamps did not flash simultaneously. We emphasize that the conclusion of nonsimultaneity relies on Einstein's second postulate. Since (as the placement of the soot marks reveals) the distances are equal, and since the light flashes travelled at the

*same* speed from the two lamps, the fact that the flashes arrived at different times means that the lamps did not explode simultaneously. We thus reach the following conclusion:

Two events that are simultaneous in one frame of reference are in general not simultaneous in a second frame moving with respect to the first; simultaneity is not an absolute concept.

In this thought experiment it is tempting to ask which observer's view of simultaneity is correct,  $O_s$ 's or  $O_m$ 's? Strangely enough they both are. Neither frame is better for judging simultaneity. Simultaneity is a relative concept rather than an absolute one. In everyday life, we are usually unaware of this effect; it becomes much more significant as the relative speed between the two observers increases to a significant fraction of *c*.

# SUMMARY

# Frames of Reference and Relativity

- Any frame of reference in which the law of inertia holds is called an inertial frame of reference.
- A noninertial frame is one that is accelerating relative to an inertial frame.
- The laws of Newtonian mechanics are only valid in an inertial frame of reference and are the same in all inertial frames of reference.
- In Newtonian mechanics, no experiment can identify which inertial frame is truly at rest and which is moving. There is no absolute inertial frame of reference and no absolute velocity.
- Michelson and Morley's interferometer experiment showed that the ether does not exist.
- The two postulates of the special theory of relativity are: (1) all laws of physics are the same in all inertial frames of reference; (2) light travels through empty space with a speed of  $c = 3.00 \times 10^8$  m/s in all inertial frames of reference.
- Simultaneity of events is a relative concept.

## Section 11.1 Questions

## **Understanding Concepts**

- 1. You are in a windowless car of a train that is either stationary or moving at a constant velocity with respect to Earth. Is there any experiment you can do in the train to determine whether you are moving? Explain your answer.
- **2.** Distinguish between an inertial and a noninertial frame of reference, and give an example of each. How do we account for motions that occur, in seeming violation of the law of inertia, in noninertial frames?
- **3.** You are travelling in a train that is slowing down upon approaching the station. You throw a heavy ball, aiming it directly at the ceiling above your chair. Relative to you, where will the ball fall? Explain your answer.

- **4.** Describe the significance of the Michelson–Morley experiment. Why was its seeming failure a success?
- **5.** State the two postulates of the special theory of relativity in such a way that they can be understood by a peer who does not study physics.
- **6.** Is there a situation where two events that occur at the same time for one observer can be simultaneous to a second observer moving with respect to the first? Explain your answer.