One of the major areas of research at the end of the nineteenth century was the spectral analysis of light emitted by hot solids and gases. In the study of blackbody radiation there was discrepancy between theory and experimental data that scientists could not reconcile. Many theories were put forward to explain and predict the details of the observed spectra, but none was adequate.

**Blackbody Radiation**

If a piece of steel is placed into the flame of a welding torch, the steel begins to glow: first dull red, then a brighter orange–red, then yellow, and finally white. At high temperatures (above 2000 K), the hot steel emits most of the visible colours of the spectrum as well as infrared radiation. If heated sufficiently, the steel can produce ultraviolet emissions as well. It has been found that this behaviour is similar for all incandescent solids, regardless of their composition. Thus, as the temperature increases, the spectrum of the emitted electromagnetic radiation shifts to higher frequencies (Figure 1). It has also been determined that the relative brightness of the different colours radiated by an incandescent solid depends mainly on the temperature of the material.

The actual spectrum of wavelengths emitted by a hot object at various temperatures is shown in Figure 2. The curves illustrate two key points:

- At a given temperature, a spectrum of different wavelengths is emitted, of varying intensity, but there is a definite intensity maximum at one particular wavelength.
- As the temperature increases, the intensity maximum shifts to a shorter wavelength (higher frequency).

The curves in Figure 2 represent the radiation from an object that approximates an ideal emitter or absorber of radiation. Such an object would absorb all wavelengths of light striking it, reflecting none. It would, therefore, appear black under reflected radiation and hence is called a blackbody. The detailed analysis of radiation absorption and emission shows that an object that absorbs all incoming radiation, of whatever wavelength, is likewise the most efficient possible emitter of radiation. The radiation emitted by a blackbody is called blackbody radiation.

Scientists in the 1890s were trying to explain the dependence of blackbody radiation on temperature. According to Maxwell’s electromagnetic theory, the radiation originates from the oscillation of electric charges in the molecules or atoms of the material.
When the temperature increases, the frequency of these oscillations also increases. The corresponding frequency of the radiated light should increase as well. According to Maxwell’s classical theory, the intensity versus wavelength must follow the dashed line in Figure 3. But this is not the case. The actual radiation follows the solid line. The region of the graph where theory and experimental data disagree is in the ultraviolet portion of the spectrum. Scientists in the 1890s referred to this problem as the ultraviolet catastrophe.

Planck’s Quantum Hypothesis

In the early 1900s, the German physicist Max Planck proposed a new, radical theory to explain the data. He hypothesized that the vibrating molecules or atoms in a heated material vibrate only with specific quantities of energy. When energy radiates from the vibrating molecule or atom, it is not emitted in a continuous form but in bundles, or packets, which Planck called quanta. He further proposed that the energy of a single quantum is directly proportional to the frequency of the radiation:

$$E = hf$$

where $E$ is the energy in joules, $f$ is the frequency in hertz, and $h$ is a constant in joule-seconds.

Planck estimated the value of the constant $h$ by fitting his equation to experimental data. Today, the constant $h$ is called Planck’s constant. Its accepted value, to three significant digits, is

**Planck’s Constant**

$$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$$

**quanta** packets of energy; one quantum is the minimum amount of energy a particle can emit

**Planck’s constant** constant with the value $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$; represents the ratio of the energy of a single quantum to its frequency
Planck further hypothesized that the emitted energy must be an integral multiple of the minimum energy, that is, the energy can only be $hf, 2hf, 3hf, \ldots$:

$$E = nhf \quad \text{where} \quad n = 1, 2, 3, \ldots$$

If light energy is quantized, and the energy of each bundle is determined by the relationship $E = hf$, the bundles in the red region will have low energy and the bundles in the ultraviolet region will have high energy. Using this quantum model and applying statistical methods beyond the level of this text, Planck explained the shape of the intensity versus wavelength graph for all areas of the spectrum, including the ultraviolet region, for a blackbody of any given temperature.

The concept of quantization can be clarified with a simple analogy: compare the change in gravitational energy as a box is pushed up a ramp, with the progress of the same box, to the same final elevation, by stairs (Figure 4). According to classical physics, the box on the ramp takes on a continuous range of gravitational potential energies as it is pushed up the ramp. According to the quantum model, the box that moves up the stairs does so in discrete, quantized “steps” of energy.

The concept of quantized values was not new. Dalton, in his theory of the atom ninety years before, had proposed that the structure of matter was based on the smallest invisible particle, the atom, and this was well accepted in 1900. Also, it had been shown by Thomson that the electric charge is quantized: the smallest charge found in nature is the charge on the electron. Nevertheless, the idea that energy is quantized was not easy to accept.

Planck’s quantum idea was revolutionary for two reasons:

- It challenged the classical wave theory of light by proposing that electromagnetic waves do not transmit energy in a continuous manner but, instead, transmit energy in small packages, or bundles.
- It challenged the classical physics of Newton, since it proposed that a physical object is not free to vibrate with any random energy; the energy is restricted to certain discrete values.

Planck, himself, was initially skeptical about his own theory. He was not ready to reject the classical theories that were so well accepted by the scientific community. He even stated that, although his hypothesis worked in explaining blackbody radiation, he hoped a better explanation would come forth.

More experimental evidence for the quantum theory was required. Quantization of energy remained generally unaccepted until 1905. In that year Einstein argued persuasively on its behalf, showing how it helped explain the photoelectric effect, as light ejects electrons from a metal in a frequency-dependent manner. (The photoelectric effect is discussed later in this chapter.) So drastic, in retrospect, was the change ushered in by Planck’s quantum hypothesis that historians established a sharp division; physics prior to 1900 became known as classical physics, and physics after 1900 was called modern physics.
physics. Since Planck’s work was so important historically, Planck’s honour and respect in the scientific community were second only to Einstein’s in the first half of the 20th century (Figure 5).

SAMPLE problem 1

Calculate the energy in joules and electron volts of
(a) a quantum of blue light with a frequency of $6.67 \times 10^{14}$ Hz
(b) a quantum of red light with a wavelength of 635 nm

Solution
(a) $f = 6.67 \times 10^{14}$ Hz
$h = 6.63 \times 10^{-34}$ J-s

$E = hf$
$= (6.63 \times 10^{-34} \text{ J-s})(6.67 \times 10^{14} \text{ Hz})$
$E = 4.42 \times 10^{-19} \text{ J}$

$1.60 \times 10^{-19} \text{ J} = 1 \text{ eV}$

$rac{4.42 \times 10^{-19} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = 2.76 \text{ eV}$

The energy is $4.42 \times 10^{-19}$ J, or 2.76 eV.

(b) $\lambda = 635 \text{ nm} = 6.35 \times 10^{-7} \text{ m}$
$h = 6.63 \times 10^{-34}$ J-s

$E = hf$

But $v = f \lambda$ or $c = f \lambda$, and $f = \frac{c}{\lambda}$, where $c$ is the speed of light $= 3.00 \times 10^8 \text{ m/s}$.

$E = \frac{hc}{\lambda}$
$= \frac{(6.63 \times 10^{-34} \text{ J-s})(3.00 \times 10^8 \text{ m/s})}{6.35 \times 10^{-7} \text{ m}}$
$E = 3.13 \times 10^{-19} \text{ J}$

$1.60 \times 10^{-19} \text{ J} = 1 \text{ eV}$

$rac{3.13 \times 10^{-19} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = 1.96 \text{ eV}$

The energy is $3.13 \times 10^{-19}$ J, or 1.96 eV.

Practice

Understanding Concepts
1. Explain which of the following quantities are discrete: time, money, matter, energy, length, scores in hockey games.
2. Determine the energy, in electron volts, for quanta of electromagnetic radiation with the following characteristics:
   (a) wavelength = 941 nm (infrared radiation)
   (b) frequency = $4.4 \times 10^{14}$ Hz (red light)
   (c) wavelength = 435 nm (violet light)
   (d) frequency = $1.2 \times 10^{18}$ Hz (X rays)

Answers
2. (a) 1.32 eV
   (b) 1.8 eV
   (c) 2.86 eV
   (d) $5.0 \times 10^3$ eV

Figure 5
Max Karl Planck (1858–1947) worked at the University of Berlin. His initial influential work on black-body radiation dates from 1889. In 1918, Planck’s discovery of energy quanta was recognized with the Nobel Prize in physics. Stimulated by Planck’s work, Einstein became an early proponent of the quantum theory. The celebrated Max Planck Society in Germany runs research institutes similar to those maintained by the National Research Council in Canada.

LEARNING TIP

Electron Volts
It is more common to use electron volts in quantum mechanics. Recall that $\Delta E = q \Delta V$; to convert joules to electron volts, use the relationship $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$.
The photoelectric effect, the phenomenon in which electrons are liberated from a substance exposed to electromagnetic radiation, is a key concept in understanding the behavior of electrons in response to light. When light strikes the photoelectric surface, electrons are ejected and flow to the collector (C). Notice that the flow of photoelectrons is opposite in direction to conventional current.

**Making Connections**

6. As you read this text, your body may be bombarded with quanta from radio waves ($\lambda = 10^2 \text{m}$) and quanta of cosmic rays, energetic particles, rather than electromagnetic waves, which nevertheless prove in quantum theory to have a wave aspect ($\lambda = 10^{-16} \text{m}$). How many quanta of radio waves would it take to impart the same amount of energy as a single quantum of cosmic radiation? Comment on the relative biological hazards posed by these two sources of energy.

**Einstein and the Photoelectric Effect**

German physicist Heinrich Hertz was testing Maxwell’s theory of electromagnetic waves in 1887 when he noticed that certain metallic surfaces lose their negative charges when exposed to ultraviolet light. He demonstrated the charge loss by wiring an insulated, polished zinc plate to a gold-leaf electroscope, as in Figure 3 in the chapter introduction. Incident ultraviolet light somehow caused the zinc plate to release electrons, as indicated by the falling leaves of the electroscope. Hertz’s phenomenon was called the photoelectric effect (since it involved both light and electricity), and the emitted electrons were called photoelectrons. Today we can easily demonstrate the photoelectric effect with visible light and a photocell (Figure 6).

Figure 7 also illustrates the photoelectric effect. A photosensitive cathode is illuminated by light of frequency $f$ and intensity $i$, causing the emission of photoelectrons from the cathode. These photoelectrons travel across the vacuum tube toward the anode, due to the applied, external potential difference, and so constitute a photocurrent, $i$, measured by the microammeter (Figure 7(a)). But the circuit also contains a variable source of electrical potential, which can make the anode negative. This has the effect of reducing the photocurrent by causing all but the faster photoelectrons to be “turned back” (Figure 7(b)). If the anode is made gradually more negative relative to the cathode, a potential difference, the cutoff potential, is reached which is just large enough to reduce the current to zero. This cutoff potential corresponds to the maximum kinetic energy of the photoelectrons.

**Answers**

3. Calculate the wavelength, in nanometres, of a quantum of electromagnetic radiation with $3.20 \times 10^{-19} \text{J}$ of energy. What colour is it? (See Section 9.6 for reference.)

4. Calculate the frequency of a 2.25-eV quantum of electromagnetic radiation.

5. Compare the respective energies of a quantum of “soft” ultraviolet radiation ($\lambda = 3.80 \times 10^{-7} \text{m}$) and a quantum of “hard” ultraviolet radiation ($\lambda = 1.14 \times 10^{-7} \text{m}$), expressing your answer as a ratio.
Many scientists repeated Hertz’s experiment with similar apparatus. Their results not only gave some support to Planck’s theories but also provided the basis for Einstein’s analysis of the photoelectric effect. You can explore the effect by performing Lab Exercise 12.1.1 in the Lab Activities section at the end of this chapter.

Here are some of the more significant findings:

1. Photoelectrons are emitted from the photoelectric surface when the incident light is above a certain frequency \( f_0 \), called the threshold frequency. Above the threshold frequency, the more intense the light, the greater the current of photoelectrons (Figure 8(a)).

2. The intensity (brightness) of the light has no effect on the threshold frequency. No matter how intense the incident light, if it is below the threshold frequency, not a single photoelectron is emitted (Figure 8(b)).

3. The threshold frequency, at which photoelectric emission first occurs, is different for different surfaces. For example, light that causes photoelectric emission from a cesium cathode has no effect on a copper cathode (Figure 9).

4. As the retarding potential applied to the anode is increased, the photocurrent \( I \) decreases, regardless of the intensity of the light. The photoelectrons are thus emitted with different kinetic energies. A value \( V_0 \) of the retarding potential is eventually reached, just sufficient to make the photocurrent zero. Even the fastest photoelectrons are now prevented from reaching the anode, being turned back by the retarding potential (Figure 10).

5. If different frequencies of light, all above the threshold frequency, are directed at the same photoelectric surface, the cutoff potential is different for each. It is found that the higher the frequency of the light, the higher the cutoff potential. The cutoff potential is related to the maximum kinetic energy with which photoelectrons are emitted: for a photoelectron of charge \( e \) and kinetic energy \( E_K \), cut off by a retarding potential \( V_0 \), \( E_K = eV_0 \). (This follows from the definition of potential difference in Chapter 7.) By illuminating several photoelectric surfaces with light of various frequencies and measuring the cutoff potential obtained for each surface, values are obtained for the graph shown in Figure 11. Although each surface has a different threshold frequency, each line has the same slope.
6. During photoemission, the release of the electron is immediate, with no appreciable delay between illumination and photoelectron emission, even for extremely weak light. It appears that the electron absorbs the light energy immediately: no time is required to accumulate sufficient energy to liberate the electrons.

Of these various experimental findings, only the first could be explained on the basis of the classical electromagnetic theory of light. In particular, according to classical wave theory, there is no reason why an intense beam of low-frequency light should not be able to produce a photoelectric current or why the intensity of the beam should not affect the maximum kinetic energy of the ejected photoelectrons. The classical wave theory of light could not properly explain the photoelectric effect at all.

As an analogy, consider a boat in a harbour. As long as the incoming water waves have a small amplitude (i.e., are of low intensity), the boat will not be tossed up on the shore, regardless of the frequency of the waves. However, according to the above observations, even a small-amplitude (low-intensity) wave can eject a photoelectron if the frequency is high enough. This is analogous to the boat being hurled up on the shore by a small wave with a high enough frequency, certainly not what classical wave theory would predict.

Einstein was well aware of these experiments and Planck’s blackbody hypothesis. He also knew about Newton’s particle theory of light, some aspects of which he resurrected to make a radical proposal: the energy of electromagnetic radiation, including visible light, is not transmitted in a continuous wave but is concentrated in bundles of energy called photons. Einstein further proposed that the energy in each of his photons was constrained to be one of a set of discrete possible values, determined by Planck’s equation, \( E = hf \). He used his photon theory of light to explain some of the experimental results of the photoelectric effect and predicted new effects as well.

Einstein reasoned that if an electron, near the surface of a metal, absorbs a photon, the energy gained by the electron might be great enough for the electron to escape from the metal. Some of the absorbed energy would be used to break away from the metal surface, with the remainder showing up as the kinetic energy of the electron (Figure 12). Since the energy of a photon is given by \( hf \), the higher the frequency, the greater the kinetic energy of the ejected photoelectron.

This behaviour also explained why there is a threshold frequency. The electron has to receive a minimum amount of energy to escape the attractive forces holding it to the metal. When the frequency of the incident light is too low, the photon does not provide the absorbing electron with sufficient energy, and it remains bound to the surface.
The intensity (brightness) of the light is only a measure of the rate at which the photons strike the surface, not of the energy per photon. This helps explain why the kinetic energy of the emitted photoelectrons and the threshold frequency are independent of the intensity of the incident light.

To summarize: when a photon hits a photoelectric surface, a surface electron absorbs its energy. Some of the energy is needed to release the electron, while the remainder becomes the kinetic energy of the ejected photoelectron. This is what one would expect based on the conservation of energy. Einstein described this mathematically as follows:

\[ E_{\text{photon}} = W + E_K \]

where \( E_{\text{photon}} \) is the energy of the incident photon, \( W \) is the energy with which the electron is bound to the photoelectric surface, and \( E_K \) is the kinetic energy of the ejected photoelectron.

Rearranging the equation, we obtain

\[ E_K = E_{\text{photon}} - W \]

Upon rewriting \( E_{\text{photon}} \) in terms of the frequency of the incident photon, we have Einstein's photoelectric equation:

\[ E_K = hf - W \]

The value \( W \) (the energy needed to release an electron from an illuminated metal) is called the work function of the metal. The work function is different for different metals (Table 1) and in most cases is less than \( 1.6 \times 10^{-18} \) J (equivalently, less than 10 eV).

When surface electrons absorb the photons, many interactions occur. Some absorbing electrons move into the surface and do not become photoelectrons at all. Others either emerge immediately at a large angle to the normal or undergo inelastic collisions before emerging. Still others emerge immediately at small angles to the normal, moving more or less directly toward the anode. The net effect is that only a small number of the more energetic photoelectrons come close to reaching the anode. They are further inhibited as the retarding voltage approaches the cutoff potential. Therefore, only the most energetic electrons reach the anode. As a result, the cutoff potential \( V_0 \) measures the maximum possible kinetic energy of the photoelectrons (represented by \( E_K \) in the equation \( E_K = hf - W \)).

The same mathematical relationship may be derived from the same three surfaces in Figure 13, except the vertical axis has been extended to include the negative intercept.

![Figure 13](image-url)

Energy of photoelectrons versus frequency of incident light for three surfaces

### Table 1

<table>
<thead>
<tr>
<th>Metal</th>
<th>Work Function (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>aluminum (Al)</td>
<td>4.20</td>
</tr>
<tr>
<td>barium (Ba)</td>
<td>2.52</td>
</tr>
<tr>
<td>cesium (Cs)</td>
<td>1.95</td>
</tr>
<tr>
<td>copper (Cu)</td>
<td>4.48</td>
</tr>
<tr>
<td>gold (Au)</td>
<td>5.47</td>
</tr>
<tr>
<td>iron (Fe)</td>
<td>4.67</td>
</tr>
<tr>
<td>lead (Pb)</td>
<td>4.25</td>
</tr>
<tr>
<td>lithium (Li)</td>
<td>2.93</td>
</tr>
<tr>
<td>mercury (Hg)</td>
<td>4.48</td>
</tr>
<tr>
<td>nickel (Ni)</td>
<td>5.22</td>
</tr>
<tr>
<td>platinum (Pt)</td>
<td>5.93</td>
</tr>
<tr>
<td>potassium (K)</td>
<td>2.29</td>
</tr>
<tr>
<td>rubidium (Rb)</td>
<td>2.26</td>
</tr>
<tr>
<td>silver (Ag)</td>
<td>4.74</td>
</tr>
<tr>
<td>sodium (Na)</td>
<td>2.36</td>
</tr>
<tr>
<td>tin (Sn)</td>
<td>4.42</td>
</tr>
<tr>
<td>zinc (Zn)</td>
<td>3.63</td>
</tr>
</tbody>
</table>

For each photoelectric surface, the graph is a straight line of the form \( y = m(x - a) \), where \( m \) is the slope and \( a \) is the horizontal intercept.

In this case,

\[ eV_0 = mf - mf_0 \]

since the intercept on the frequency-axis is \( f_0 \).

But we have already stated that \( eV_0 \) is equal to the kinetic energy of the emitted photoelectron. Thus, the equation becomes

\[ E_K = mf - mf_0 \]

which matches Einstein’s equation if \( m \), the slope of the line on the graph, equals Planck’s constant \( h \), and \( W = hf_0 \).

Further, if the straight line is extended (dashed lines in Figure 13) until it crosses the vertical axis, the equation \( E_K = hf - W \) is seen to be of the form \( y = mx + b \), with the absolute value of the vertical intercept \( b \) giving the work function \( W \) for that surface.

Note that on a graph of \( E_K \) (or \( eV_0 \)) versus \( f \), each photosurface has the same slope (\( h \), or Planck’s constant), but each has its own unique horizontal intercept (\( f_0 \), or threshold frequency) and vertical intercept (\( -W \), or work function) (Figure 14).

**Figure 14**

For sodium, the threshold frequency \( f_0 \) is \( 6.0 \times 10^{14} \) Hz, and the work function \( W \) is \(-2.5\) eV. These values represent the intercepts on the frequency and energy axes, respectively.

Einstein’s photoelectric equation agreed qualitatively with the experimental results available in 1905, but quantitative results were required to find out if the maximum kinetic energy did indeed increase linearly with the frequency and if the slope of the graph, \( h \), was common to all photoelectric substances.

Oxidization impurities on the surfaces of metal photocathodes caused difficulties for the experimentalists. It was not until 1916 that Robert Millikan designed an apparatus in which the metallic surface was cut clean in a vacuum before each set of readings. Millikan showed Einstein’s explanation and prediction to be correct. Millikan also found the numerical value \( h \), as the slope of his plots, to be in agreement with the value Planck had calculated earlier, using a completely different method. It was for his explanation of the photoelectric effect, based on the photon theory of light, that Albert Einstein received the 1921 Nobel Prize in physics (not, as is sometimes claimed, for his still more celebrated work on relativity).
Orange light with a wavelength of $6.00 \times 10^2$ nm is directed at a metallic surface with a work function of 1.60 eV. Calculate

(a) the maximum kinetic energy, in joules, of the emitted electrons
(b) their maximum speed
(c) the cutoff potential necessary to stop these electrons

**Solution**

(a) $\lambda = 6.00 \times 10^2$ nm $=$ $6.00 \times 10^{-7}$ m

$h = 6.63 \times 10^{-34}$ J-s

$E_K =$ ?

$W = 1.60 \text{ eV}$

$= (1.60 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})$

$W = 2.56 \times 10^{-19} \text{ J}$

$E_{K,\text{max}} = \frac{hc}{\lambda} - W$

$= \frac{(6.63 \times 10^{-34} \text{ J-s})(3.00 \times 10^8 \text{ m/s})}{6.00 \times 10^{-7} \text{ m}} - 2.56 \times 10^{-19} \text{ J}$

$E_{K,\text{max}} = 7.55 \times 10^{-20} \text{ J}$

The maximum kinetic energy of the emitted photons is $7.55 \times 10^{-20}$ J.

(b) $v =$ ?

$m_e = 9.11 \times 10^{-31}$ kg (from Appendix C)

$E_K = \frac{1}{2}m v^2$

$v = \sqrt{\frac{2E_K}{m}}$

$= \sqrt{\frac{2(7.55 \times 10^{-20} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}}$

$v = 4.07 \times 10^5 \text{ m/s}$

The maximum speed of the emitted electrons is $4.07 \times 10^5 \text{ m/s}$.

(c) $V_0 =$ ?

$E_K = eV_0$

$V_0 = \frac{E_K}{e}$

$= \frac{7.55 \times 10^{-20} \text{ J}}{1.60 \times 10^{-19} \text{ C}}$

$V_0 = 0.472 \text{ V}$

The cutoff potential necessary to stop these electrons is 0.472 V.

The answer to (c) could also have been determined from (a), as follows:

$7.55 \times 10^{-20} \text{ J} = \frac{7.55 \times 10^{-20} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = 0.472 \text{ eV}$
Photodiodes and Digital Cameras

In addition to its theoretical role in confirming the photon view of light, the photoelectric effect has many practical applications, for the most part in semiconductors known as photodiodes. The absorption of a photon liberates an electron, which changes the conductivity of the photodiode material. Burglar alarms, garage doors, and automatic door openers often incorporate photodiodes. When a beam of light is interrupted, the drop in current in the circuit activates a switch, triggering an alarm or starting a motor. An infrared beam is sometimes used because of its invisibility. Remote controls for some televisions and video machines work in much the same way. Many smoke detectors use the photoelectric effect: the particles of smoke interrupt the flow of light and alter the electric current. Photocell sensors in cameras measure the level of light, adjusting the shutter speed or the aperture for the correct exposure. Photocells are used in a host of other devices, such as outdoor security lights and automatic street lights.

An important application of the photoelectric effect is the charge-coupled device (CCD). An array of these devices is used in digital cameras to capture images electronically.

A CCD array is a sandwich of semiconducting silicon and insulating silicon dioxide, with electrodes. The array is divided into many small sections, or pixels, as in Figure 15 (where, for simplicity, we show a 16-pixel array, even though some cameras’ arrays have over three million). The blow-up in Figure 15 shows a single pixel. Incident photons of visible light strike the silicon, liberating photoelectrons as a result of the photoelectric effect. The number of electrons produced is proportional to the number of photons striking the pixel. Each pixel in the CCD array thus accumulates an accurate representation of the light intensity at that point on the image. In some applications, prisms or red, green, or blue filters are used to separate the colours.
The CCD was invented in 1969 by Willard S. Boyle (Figure 16), a Canadian, and George Smith, at Bell Research Laboratories in the United States. Their invention has not only brought digital photography, including video recording, to the consumer marketplace, but has triggered a revolution in astronomical image-processing.

**Practice**

**Making Connections**

17. The photoelectric effect has many applications. Choose one, either from this text or from a search of the Internet or other media. Prepare a research paper, using the following as a guide:

(a) Explain in detail, with the help of labelled diagrams, how your chosen device detects light using the photoelectric effect.

(b) Explain how your device uses information from the photoelectric detector.

(c) Identify at least three other devices that operate in a similar manner.

---

**Momentum of a Photon: The Compton Effect**

In 1923, the American physicist A.H. Compton (1892–1962) directed a beam of high-energy X-ray photons at a thin metal foil. The experiment was similar to the photoelectric experiments, except that high-energy X-ray photons were used instead of light. Compton not only observed ejected electrons, as was to be expected from the theory of the photoelectric effect, but also detected an emission of X-ray photons, lower in energy, and therefore lower in frequency, than the photons in the bombarding beam. He also noted that electrons were scattered at an angle to the X-ray photons. This scattering of lower-frequency X-ray photons from foil bombarded by high-energy photons is known as the **Compton effect** (Figure 17).

A whole series of experiments, using different metal foils and different beams of X rays, produced similar results that could not be explained using electromagnetic-wave theory. Compton proposed that the incident X-ray photon acts like a particle that collides elastically with an electron in the metal, emerging with lower energy. The electron flies off with the kinetic energy it gained in the collision. Compton’s data indicated that energy was indeed conserved.

If energy were conserved in the collision (see Figure 18), the following would be true:

\[
E_{\text{X-ray}} = E'_{\text{X-ray}} + E_{\text{electron}}
\]

\[
hf = hf' + \frac{1}{2}mv^2
\]

Compton’s stroke of genius was to inquire whether momentum is conserved, as in ordinary collisions. It is not, at first, clear in what sense momentum could be associated with a bundle of energy with no mass, travelling at the speed of light. Compton solved this problem by using Einstein’s \( E = mc^2 \) equation from special relativity. A body with energy \( E \) has a mass equivalence of \( \frac{E}{c^2} \) (see Section 11.3). Compton’s solution was the following: the magnitude of the momentum \( p \) of a body is defined as the product of mass \( m \) and speed \( v \), \( p = mv \). If we replace \( m \) with its mass equivalent, \( \frac{E}{c^2} \), and replace \( v \) with \( c \), we can write

\[ p = \left( \frac{E}{c^2} \right) v \quad \text{or} \quad p = \frac{E}{c} \]

as an expression for \( p \) in which mass does not explicitly appear.
Now from the photon’s energy \( E = hf \) and the universal wave equation \( c = f \lambda \), \( E \) and \( c \) are replaced to give \( p = \frac{hf}{f \lambda} \), or

\[
p = \frac{h}{\lambda}
\]

This relation gives the magnitude of the momentum of a photon, where \( p \) is the magnitude of the momentum, in kilogram-metres per second, \( h \) is Planck’s constant with a value of \( 6.63 \times 10^{-34} \) J·s, and \( \lambda \) is the wavelength, in metres.

Using this definition for photon momentum, Compton found that the conservation of momentum did hold for X-ray scattering collisions, as seen in Figure 19 and expressed in the vector equation

\[
\vec{p}_{\text{photon}} = \vec{p}_{\text{photon}}' + \vec{p}_{\text{electron}}
\]

Compton’s experiments clearly demonstrated the particle-like aspects of light, for not only can a discrete energy, \( hf \), be assigned to a photon, but also a value of momentum, \( \frac{h}{\lambda} \). His work provided conclusive evidence for the photon theory of light. As a result, Compton was awarded the Nobel Prize for physics in 1927.

### DID YOU KNOW? Photon

The word “photon” is similar to the names given to various other particles: electron, proton, neutron.

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**SAMPLE problem 3**

What is the magnitude of the momentum of a photon with a wavelength of \( 1.2 \times 10^{-12} \) m?

**Solution**

\[
\lambda = 1.2 \times 10^{-12} \text{ m} \\
h = 6.63 \times 10^{-34} \text{ J·s} \\
p = ?
\]

\[
p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ J·s}}{1.2 \times 10^{-12} \text{ m}} = 5.5 \times 10^{-22} \text{ kg·m/s}
\]

The magnitude of the momentum of the photon is \( 5.5 \times 10^{-22} \) kg·m/s.
Interactions of Photons with Matter

If an intense beam of light is directed at the surface of an absorbing material, the energy of the photons is mostly absorbed by that surface. As a result, the surface heats up. But the Compton effect shows that photons transfer momentum as well. The sum of the impacts on the surface of all of the photons per unit of time results in pressure on the surface. This pressure is not normally discernible. (We do not feel the pressure of light when we walk out into sunlight or stand under a strong lamp.) Today, however, using very sensitive equipment, we can actually measure the pressure of light on a surface and confirm that the relationship \( p = \frac{h}{\lambda} \) is a valid expression for the momentum of an individual photon.

We have seen, with both the photoelectric effect and the Compton effect, that when a photon comes into contact with matter, there is an interaction. Five main interactions are possible:

1. The most common interaction is simple reflection, as when photons of visible light undergo perfectly elastic collisions with a mirror.
2. In the case of the photoelectric effect, a photon may liberate an electron, being absorbed in the process.
3. In the Compton effect, the photon emerges with less energy and momentum, having ejected a photoelectron. After its interaction with matter, the photon still travels at the speed of light but is less energetic, having a lower frequency.
4. A photon may interact with an individual atom, elevating an electron to a higher energy level within the atom. In this case, the photon completely disappears. All of its energy is transferred to the atom, causing the atom to be in an energized, or “excited,” state. (We will examine the details of this interaction later in this chapter.)
5. A photon can disappear altogether, creating two particles of nonzero mass in a process called pair production (see Section 13.1). Pair production requires a photon of very high energy (> 1.02 MeV) and, correspondingly, a very short wavelength (as with X-ray and gamma-ray photons). When such a photon collides with a heavy nucleus, it disappears, creating an electron (\( \beta^- \)e) and a particle of equal mass but opposite charge, the positron (\( \beta^+ \)e) (Figure 20). This creation of mass from energy obeys Einstein’s mass–energy equivalence equation \( E = mc^2 \).
Section 12.1 Questions

Understanding Concepts

1. List the historical discoveries and interpretations that led to the confirmation of the photon theory of light.

2. Use Planck's quantum theory to suggest a reason why no photoelectrons are released from a surface until light of sufficiently high frequency is incident on the surface.

3. List at least five devices in your home that operate using the photoelectric effect.

4. State which physical quantities are represented by the symbols in the photoelectric equation \( E_K = hf - W \).

5. Compare and contrast the photoelectric effect and the Compton effect.

6. Describe five interactions between photons and matter.

7. The average temperature in the visible incandescent layers of the Sun is about 6000 K. An incandescent light has a temperature of 2500 K.
   (a) Why do artificial sources of light not provide proper illumination when exposing a colour film designed to be used in sunlight?
   (b) Why does a xenon flash (operating at about 6000 K) provide proper illumination for the same daylight film?

8. Calculate the energy of an ultraviolet photon, of wavelength 122 nm, in both joules and electron volts.
9. The relationship $E = \frac{1.24 \times 10^3}{\lambda}$, where $E$ is the energy of a photon in electron volts and $\lambda$ is its wavelength in nanometres, is a handy form of the equation $E = \frac{hc}{\lambda}$. The relationship is quite useful in quantum-mechanics calculations, since many measurements with subatomic particles and photons involve electron volts and nanometres. Substitute in $E = \frac{hc}{\lambda}$, and make any necessary unit conversions, to show that $E = \frac{1.24 \times 10^3}{\lambda}$ is valid for nanometres and electron volts.

10. How does the pair of curves in Figure 8(b) show that the maximum speed of the photoelectrons is independent of the intensity of the light directed at the photoelectric surface?

11. Locate sodium and copper in Table 1. Does it take more energy to remove a photoelectron from sodium or from copper? Which has the higher threshold frequency, potassium or barium? Explain your answers.

12. Find the minimum frequency of the light required to eject photoelectrons from a metallic surface whose work function is $7.2 \times 10^{-19}$ J.

13. What wavelength of light is required for ejecting photoelectrons from a tungsten surface $(W = 4.52 \text{ eV})$ if the maximum kinetic energy of the electrons is $1.68 \text{ eV}$?

14. When light of wavelength of 482 nm falls onto a certain metallic surface, a retarding potential of 1.2 V proves just sufficient to make the current passing through the phototube fall to zero. Calculate the work function of the metal.

15. (a) Calculate the frequency of a photon whose wavelength is $2.0 \times 10^{-7}$ m.
(b) Calculate the energy of the same photon, in both joules and electron volts.
(c) Calculate the momentum of the same photon.

16. (a) Calculate the momentum of a photon of wavelength $2.50 \times 10^{-9}$ m.
(b) Calculate the speed of an electron having the same momentum as the photon in (a). $(m_e = 9.11 \times 10^{-31}$ kg$)$.
(c) Calculate the kinetic energy of the electron. How does it compare with the energy of the photon?

17. Calculate, in electron volts, the energies of the photons emitted by radio stations of frequencies $5.70 \times 10^2$ kHz and $102$ MHz.

**Applying Inquiry Skills**

18. Photosynthesis is the chemical change of carbon dioxide and water into sugar and oxygen, in the presence of light and chlorophyll. The graph in Figure 21 shows that the rate of the reaction depends on the wavelength of the incident light. Using the photon theory and the graph, explain

(a) why leaves containing chlorophyll appear green in white light
(b) why pure green light does not produce photosynthesis
(c) why an incandescent lamp, operating at 2500 K, does not produce sufficient photosynthesis to maintain the health of a green plant

![Rate of Photosynthesis](Image)

**Figure 21**
Rate of photosynthesis versus wavelength of incident light

**Making Connections**

19. Suggest some ways life would be different if Planck’s constant were much larger or much smaller than its actual value.

20. A certain solar-powered calculator receives light with an average wavelength of 552 nm, converts 15% of the incoming solar energy into electrical energy, and consumes 1.0 mW of power. Calculate the number of light quanta needed by the calculator each second.

21. (a) Calculate the energy of a single microwave quantum of wavelength of 1.0 cm.
(b) Calculate how many quanta of 1.0-cm microwave energy would be required to raise the temperature of 250.0 mL of water from 20°C to the boiling point, given that the specific heat capacity of water is $4.2 \times 10^3$ J/kg°C.

22. Research the Internet to learn how astronomers use photodetectors on NASA’s Hubble Space Telescope to produce galactic images such as the one at the beginning of Chapter 9.

23. Ultraviolet light can kill skin cells, as it does when you are sunburned. Infrared light, also from the Sun, only warms skin cells. Explain this difference in behaviour using the photon theory.